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THE DEVELOPMENT OF A MATHEMATICAL CONCEPT TO INTERPRET THE RELATION BETWEEN PLANT COMPOSITION AND CROP YIELD

Bernardo G. Capó and George Samuels[1]

INTRODUCTION

The hypothesis that a relationship exists between the composition of a plant and its yield is a valid one which has gained acceptance in the world of science. The mechanism and values of this relationship have been proposed and discussed by such workers as Borden (1–3)[2], Clements (6), Innes (8), Macy (11), Mitchell and Chandler (12), Thomas (14), Tyner (15), and Ulrich (16). However, the expression of this relationship in mathematical form is not readily accomplished.

Borden, working with sugarcane in Hawaii (1–3), made use of certain critical values for the percentage of major nutrients in the leaf and cane at various sampling periods that were associated with optimum yields, but he did not designate any specific mathematical relationship between plant composition and yield. Clements (6), also working with sugarcane in Hawaii, designated the "primary index" or percentage of total sugars in leaf sheaths as a criterion of when to fertilize. Innes (8), in Jamaica, found a linear relation between the relative increases in percentage of nutrients in the leaves, on a dry-matter basis, and the relative response in yield. The general equation determined was that the relative response in yield equalled a constant times the relative increase in percentage of leaf nutrient, on a *dry-matter* basis—plus another constant.

Macy (12), using the concept of a critical content of a nutrient by a plant, above which luxury consumption and below which poverty adjustment were assumed, presented no definite mathematical relationship between plant yield and composition. Thomas (14) developed the concept of foliar diagnosis for following the "course of nutrition" as reflected by "intensity" and "quality". He, however, made no attempt to evolve this concept into a mathematical form. Tyner (15) obtained a highly significant

[1] Associate Director for Research, and Plant Physiologist, respectively, Agricultural Experiment Station, University of Puerto Rico, Río Piedras, P. R.

[2] Numbers in parentheses refer to Literature Cited, pp. 263–4.

correlation for a linear relation between yield and nutrient content of the leaves of corn. Ulrich (16) expressed the general relationship of plant-nutrient composition in a form similar to Jenny's (9) equation for the factors affecting soil formation. In an equation for plant nutrients, Ulrich stated that the concentration of a given nutrient X would be a function of the soil S, climate Cl, time T, plant P, management M, and possibly others. This relationship was expressed in the following generalized equation:

$$X = f\ (S,\ Cl,\ T,\ P,\ M,\ \cdots).$$

Ulrich did not offer any specific equation for relating the yield of the plant to its composition.

It is the purpose of this paper to suggest a mathematical relationship as an approximation to the quantitative relationship existing between the composition of a plant and its yield as determined for hegari sorghum in the greenhouse under varying climatic and soil conditions, and fertilizer levels.

EXPERIMENTAL PROCEDURE

The data to test the relationship between plant composition and yield were obtained from pot tests performed by Capó (4) to determine the available nutrient contents of Puerto Rican soils. The details of the greenhouse technique and procedure have been described in a previous publication (5).

For the purposes of this study the harvested hegari sorghum plants used in some of the experiments were analyzed for nitrogen, phosphorus, and potassium. A total of 78 pairs of corresponding values for nitrogen, 73 for phosphorus, and 43 for potassium were utilized; all treatments were replicated from two to four times. The experiments used in this study were conducted on samples of 28 different soil series. The levels of nitrogen, phosphorus, and potassium used, together with the soil-to-sand dilutions, are given in table 1. The mean yields of sorghum and its content of nitrogen, phosphorus, and potassium, are also given in table 1. The experiments were so designed that when any one fertilizer element was varied, (nitrogen, phosphorus, or potassium), the remaining two elements were supplied in quantities sufficient to meet all nutrient demands, so as not to be limiting. For each nutrient studied there existed three levels of the nutrient: none, one unit, and two units (see table 1).

ARC TANGENT AND PLANT YIELDS

As the soil pot tests used for the experimental data were conducted using various soils with different soil-to-sand dilutions, and were carried out in different seasons of the year, the comparison of yields from different experiments was invalid on a direct basis. However, the various experiments

TABLE 1.—*The results of Mitscherlich pot tests involving nitrogen, phosphorus, and potassium on the soils of Puerto Rico with hegari sorghum as the indicator crop*

Experiment No.	Soil[1]	Fertilizer units[2]	Mean yield of dry matter per pot	Relative yield (*Yr*)	Nutrient content of plant in dry weight	Arc tangent of nutrient content (*X*)
		A: *Nitrogen*				
			Grams		*Percent*	
35	Cialitos clay	0	8.45	20.4	0.48	25.6
		1	42.35	102.4	.81	39.0
		2	41.35	100.0	1.49	56.2
36	Coloso silt loam	0	12.15	27.4	.55	28.8
		1	33.10	74.6	.79	38.3
		2	44.35	100.0	1.26	51.9
37	Catalina clay (level phase)	0	14.40	37.5	.61	31.4
		1	33.55	87.3	.94	43.2
		2	38.45	100.0	1.64	58.6
38	Vega Alta sandy loam	1	31.05	98.7	1.15	49.0
		2	31.45	100.0	1.82	61.2
39	Múcara silty clay loam	1	37.20	82.9	.81	39.0
		2	44.85	100.0	1.41	54.6
40	Ciales clay loam	1	38.90	106.4	.92	42.6
		2	36.55	100.0	1.50	56.3
41	Tiburones muck	1	15.80	74.4	1.73	60.0
		2	21.55	100.0	1.73	60.0
42	Los Guineos clay	0	19.10	40.9	.62	31.8
		1	36.50	78.1	1.03	45.9
		2	46.75	100.0	1.48	56.0
43	Toa silty clay loam	0	4.70	13.0	.67	33.8
		1	27.40	75.8	.61	31.4
		2	36.15	100.0	1.18	49.7
44	Nipe clay	0	5.20	51.2	.61	31.4
		1	19.20	189.2	1.13	48.5
		2	10.15	100.0	1.98	63.2
45	Coloso clay	0	8.85	20.9	.62	31.8
		1	34.35	81.2	.78	38.0
		2	42.30	100.0	1.29	52.2
46	Cabo Rojo clay	0	6.60	24.9	.63	32.2
		1	18.00	67.8	1.17	49.5
		2	26.55	100.0	1.74	60.1
47	Mabí clay	0	14.90	32.1	.46	24.7
		1	52.25	112.7	.68	34.2
		2	46.35	100.0	1.20	50.1
48	Múcara silt loam	0	19.15	30.9	.59	30.5
		1	47.15	76.2	.56	29.2
		2	61.90	100.0	.65	33.0
64	Aguilita stony clay	0	10.55	62.1	.75	36.9
		1	15.55	91.5	1.13	48.5
		2	17.00	100.0	1.52	56.7

TABLE 1.—*Continued*

Experiment No.	Soil[1]	Fertilizer units[2]	Mean yield of dry matter per pot	Relative yield (Yr)	Nutrient content of plant in dry weight	Arc tangent of nutrient content (X)
	A: *Nitrogen—Continued*					
			Grams		*Percent*	
65	Ponceña clay	0	9.20	48.2	0.81	39.0
		1	17.40	91.1	1.12	48.2
		2	19.10	100.0	2.21	65.6
66	Amelia clay	0	3.00	23.2	1.30	48.5
		1	9.70	74.9	1.33	53.1
		2	12.95	100.0	1.52	56.7
67	Guánica clay	0	16.65	100.6	1.21	50.4
		1	18.45	111.5	1.47	55.8
		2	16.75	100.0	1.49	56.1
68	Fraternidad clay	0	7.65	46.8	.86	40.7
		1	11.30	69.1	1.05	46.4
		2	16.70	100.0	1.49	56.1
69	Mercedita clay	0	7.55	32.3	.68	34.2
		1	13.50	57.7	.86	40.7
		2	21.60	100.0	1.05	46.4
76	Moca clay	0	7.80	98.4	3.28	73.0
		2	7.93	100.0	2.85	70.7
77	Colinas clay	0	4.87	45.6	1.02	45.6
		2	13.50	100.0	2.09	64.4
78	Vega Baja silty clay	0	7.88	53.1	.64	32.6
		1	11.40	76.8	.89	41.7
		2	14.85	100.0	1.59	57.8
80	Aguirre clay	0	7.05	44.6	.62	31.8
		1	10.85	68.7	.82	39.4
		2	15.80	100.0	1.21	50.4
81	Vayas clay	0	8.40	34.9	.62	31.8
		1	17.45	72.6	.86	40.7
		2	24.05	100.0	1.25	51.3
82	Mabí clay	0	10.75	55.6	.74	36.5
		1	18.15	93.8	.86	40.7
		2	19.35	100.0	1.30	52.5
83	Aguirre clay	0	5.40	39.9	.68	34.2
		1	13.40	98.9	.58	30.1
		2	13.55	100.0	.98	44.4
88	Fajardo clay	0	9.98	70.3	1.54	57.0
		1	13.48	94.9	1.89	62.1
		2	14.20	100.0	2.16	65.2
	B: *Phosphorus*					
35	Cialitos clay	1	26.60	64.3	0.27	15.1
		2	41.35	100.0	.39	21.1

TABLE 1.—*Continued*

Experiment No.	Soil[1]	Fertilizer units[2]	Mean yield of dry matter per pot	Relative yield (Yr)	Nutrient content of plant in dry weight	Arc tangent of nutrient content (X)
B: *Phosphorus—Continued*						
			Grams		*Percent*	
36	Coloso silt loam	0	15.80	35.6	0.42	22.8
		1	40.65	91.7	.51	27.0
		2	44.35	100.0	.33	18.4
37	Catalina clay (level phase)	1	26.95	70.1	.41	22.3
		2	38.45	100.0	.39	21.1
38	Vega Alta sandy loam	0	1.95	6.2	.16	9.1
		1	30.20	96.0	.28	15.6
		2	31.45	100.0	.37	20.3
39	Múcara silty clay loam	0	2.80	6.2	.18	10.2
		1	32.45	72.4	.28	15.6
		2	44.85	100.0	.30	16.7
40	Ciales clay loam	0	14.40	21.8	.35	10.2
		1	38.90	86.8	.30	17.7
		2	36.55	100.0	.49	26.1
41	Tiburones muck	1	22.15	104.0	.43	23.3
		2	21.25	100.0	.60	31.0
42	Los Guineos clay	1	36.50	78.1	.14	8.0
		2	46.75	100.0	.24	13.5
43	Toa silty clay loam	1	27.40	75.8	.32	16.7
		2	36.15	100.0	.31	17.2
45	Coloso clay	1	34.25	81.2	.28	16.7
		2	42.30	100.0	.34	18.7
46	Cabo Rojo clay	1	18.00	67.8	.27	16.2
		2	26.55	100.0	.30	16.7
47	Mabí clay	1	52.25	112.7	.25	14.0
		2	46.35	100.0	.25	14.1
48	Múcara silt loam	0	24.80	39.0	.27	14.0
		1	58.55	92.0	.26	15.1
		2	63.65	100.0	.26	20.3
64	Aguilita stony clay	0	8.30	61.3	.31	14.6
		1	12.75	94.1	.38	17.2
		2	13.55	100.0	.43	23.3
65	Ponceña clay	0	18.85	118.6	.70	35.0
		1	21.55	135.5	.64	32.6
		2	15.90	100.0	.67	33.7
66	Amelia clay	0	3.90	29.9	.25	14.0
		1	9.25	70.9	.47	25.2
		2	13.05	100.0	.52	27.4
67	Guánica clay	0	17.50	105.7	.47	25.2
		1	20.35	123.0	.59	30.5
		2	16.75	100.0	.70	35.0

TABLE 1.—*Continued*

Experiment No.	Soil[1]	Fertilizer units[2]	Mean yield of dry matter per pot	Relative yield (Yr)	Nutrient content of plant in dry weight	Arc tangent of nutrient content (X)
	B: *Phosphorus—Continued*					
			Grams		*Percent*	
68	Fraternidad clay	0	14.25	87.2	0.50	26.6
		1	15.05	92.0	.55	28.8
		2	16.70	100.0	.59	30.6
69	Mercedita clay	0	16.55	70.7	.36	19.8
		1	23.05	98.5	.49	26.1
		2	23.60	100.0	.51	27.1
76	Moca clay	0	2.40	25.6	.38	20.8
		2	9.37	100.0	.34	18.7
77	Colinas clay	0	3.93	28.1	.15	8.5
		2	14.00	100.0	.50	26.6
78	Vega Baja silty clay	0	7.87	52.7	.52	27.5
		1	11.90	80.1	.40	21.8
		2	14.85	100.0	.42	22.8
80	Aguirre clay	0	17.10	108.3	.30	16.7
		1	17.05	107.9	.33	18.3
		2	15.80	100.0	.47	25.2
81	Vayas clay	0	20.00	83.2	.48	25.6
		1	17.40	72.3	.77	37.6
		2	24.05	100.0	1.07	47.0
82	Mabí clay	0	18.55	95.9	.67	33.8
		1	19.50	100.8	.86	40.7
		2	19.35	100.0	.89	41.7
83	Aguirre clay	0	11.55	85.2	.41	22.3
		1	14.70	108.5	.35	19.3
		2	13.55	100.0	.51	27.1
88	Fajardo clay	0	8.02	62.6	.37	20.3
		2	12.82	100.0	.42	22.8
V	Vega Baja silty clay	0	12.90	76.1	.10	5.7
		1	14.38	84.8	.29	16.2
		2	16.95	100.0	.29	16.1
	C: *Potassium*					
48	Múcara silt loam	0	62.95	96.3	0.95	43.5
		1	53.30	81.6	1.61	58.2
		2	65.35	100.0	1.60	58.0
64	Aguilita stony clay	0	12.05	82.3	2.12	64.7
		1	18.85	128.7	2.54	68.5
		2	14.65	100.0	2.18	64.5
65	Ponceña clay	0	23.35	102.6	1.66	58.9
		1	21.10	92.7	2.31	66.6
		2	22.75	100.0	2.14	64.9

TABLE 1.—*Continued*

Experiment No.	Soil[1]	Fertilizer units[2]	Mean yield of dry matter per pot	Relative yield (Yr)	Nutrient content of plant in dry weight	Arc tangent of nutrient content (X)
	C: *Potassium—Continued*					
			Grams		*Percent*	
66	Amelia clay	0	6.90	54.5	1.10	47.7
		1	8.85	70.0	1.74	60.1
		2	12.65	100.0	2.04	63.9
67	Guánica clay	0	21.00	120.7	3.08	72.0
		1	17.35	99.7	3.30	73.1
		2	17.40	100.0	3.10	72.1
68	Fraternidad clay	0	13.40	77.5	1.67	59.1
		1	13.75	79.5	2.40	67.4
		2	17.30	100.0	2.52	68.4
69	Mercedita clay	0	17.65	81.9	2.51	68.3
		1	20.05	93.0	2.55	68.6
		2	21.55	100.0	2.95	71.3
76	Moca clay	0	9.00	95.7	1.86	61.7
		2	9.40	100.0	2.06	64.1
77	Colinas clay	0	11.03	72.4	1.86	47.5
		2	15.23	100.0	2.06	64.1
78	Vega Baja silty clay	0	12.45	83.8	1.69	59.4
		1	14.28	96.2	1.20	50.2
		2	14.85	100.0	1.25	51.4
80	Aguirre clay	0	11.05	70.0	1.25	51.3
		1	17.70	112.1	1.71	59.7
		2	15.80	100.0	1.80	60.9
81	Vayas clay	0	16.95	70.5	1.38	54.1
		1	19.85	82.5	1.79	60.8
		2	24.03	100.0	2.03	63.8
82	Mabí clay	0	17.50	90.4	1.17	49.5
		1	18.25	94.3	1.76	60.4
		2	19.35	100.0	2.58	68.8
83	Aguirre clay	0	10.20	75.3	1.62	58.3
		1	15.50	114.8	1.66	58.9
		2	13.35	100.0	1.79	60.8
88	Fajardo clay	0	13.58	91.6	1.28	52.0
		1	14.48	97.7	1.92	62.5
		2	14.82	100.0	2.20	65.5

[1] Soil: sand mixture was 1:2 for all experiments with nitrogen; for phosphorus 1:2 for experiments 35–47, 67–69, 78–83, and V; 1:1 for experiments 48, 64–66, 76, 77, and 88; for potassium 1:2 for experiments 78–83; 1:5 for experiments 48, 64–66, and 88; and 1:6 for experiments 67–77.

[2] Fertilizer units: 1N = 0.5 gm. NH_3 per pot, 1P = 0.5 gm. P_2O_5 per pot, and 1K = 0.5 gm. K_2O per pot for experiments 35, 47, 76, 77; 1N = 0.3 gm. NH_3 per pot, 1P = 0.3 gm. per pot, and 1K = 0.3 gm. K_2O per pot for experiments 48, 64–69, 78–88. When one fertilizer element was varied, the other two were kept at the highest (2 fertilizer units) level.

could be compared by means of "relative yields". The relative yield was defined for this purpose as the percentage which the actual yield of a crop was of the yield obtained with the heaviest application of the given nutrient made to the growth medium.

The relative yield for the series of experiments was determined by use of the formula:

$$Yr = \frac{Ya}{Yn} \times 100, \qquad 1$$

where Ya is the yield of the treatment in which there is zero or increasing increments of a given fertilizer element, Yn is the yield of the treatment in which the given fertilizer element is applied in maximum quantity, and Yr is the relative yield. The relative yields of the pot tests are given in table 1, by treatments.

To evaluate the relationship between the nutrient composition of the sorghum plants and their yields several relationships were tried. Among these, the one that worked best was:

$$Yr = A + B \textit{ arc tan percent } Nu, \qquad 2$$

where Yr is the relative yield, A and B are constants, and *arc tan percent Nu* is the arc whose tangent is the percentage of the respective nutrient in the plant on a dry-matter basis.

Equation 2 represents a straight-line relationship between relative yields and arc-tangent percentages. It can accordingly be written as

$$Yr = A + BX, \qquad 3$$

where X = *arc tan percent Nu*.

To fit equation 3 to the data, the percentage nutrient contents were, therefore, transformed into the arcs or angles with tangents equivalent to the given percentage contents. These angles were expressed in degrees. The fitting of this equation was then performed by calculating constants A and B according to the usual method of curve-fitting based on the principle of least squares.

According to this method,

$$B = \frac{SXYr - (SX)(SYr)/n}{SX^2 - (SX)^2/n} \qquad 4$$

and

$$A = SYr/n - BSX/n, \qquad 5$$

where SX = sum of values of X,
SYr = sum of values of Yr,
SX^2 = sum of squares of values of X,
$SXYr$ = sum of products of the corresponding values of X and Yr,
and n = total number of corresponding pairs of X and Yr values.

The values of A and B were calculated to fit the corresponding equations for nitrogen, phosphorus, and potassium to the data of table 1.

Nitrogen

$$n = 78; SX = 3{,}586.6; SYr = 6{,}154.5; S(XYr) = 300{,}108.7;$$

$$SX^2 = 175{,}504.6; SYr^2 \doteq 557{,}563.0$$

$$B = \frac{S(XYr) - (SX)(SYr)/n}{SX^2 - (SX)^2/n} = \frac{300{,}108.7 - 282{,}996.5}{175{,}504.6 - 164{,}919.2} = 1.617$$

$$A = SYr/n - BSX/n = 6{,}154.7/78 - 1.617\ (3{,}586.6/78) = 4.56$$

$$Yr = A + BX = 4.56 + 1.617X_n$$ [3]

The statistical validity of this equation was determined by calculating the significance of the regression by the usual statistical techniques as follows:

Total sum of squares $= SYr^2 - (SYr)^2/n = 557{,}563.0 - (6{,}154.5)^2/78 = 71{,}949.3$

Reduction in sum of squares due to regression $= B\ [SXYr - (SX)(SYr)/n] = 1.617(17{,}112.2) = 27{,}670.4$

Reduced sum of squares = total sum of squares − reduction in sum of squares $= 71{,}949.3 - 27{,}670.4 = 44{,}278.9$

Reduced variance = reduced sum of squares/$(n - 2) = 44{,}278.9/76 = 582.6$

F value for significance of regression $= 27{,}670.4/582.6 = 47.50$

The regression of relative yield on arc tangent percent nitrogen is significant at the 1-percent level.

Similar calculations were made for phosphorus and potassium.

Phosphorus

$$n = 73; SX = 1{,}610.0; SYr = 6{,}182.6; S(XYr) = 143{,}939.9;$$

$$SX^2 = 40{,}319.1; SYr^2 = 575{,}754.2$$

$$B = \frac{143{,}939.9 - (1{,}610.0)(6{,}182.6)/73}{40{,}319.1 - (1{,}610.0)^2/73} = 1{,}576$$

$$A = 6{,}182.6/73 - 1.576(1{,}610.0/73) = 49.94$$

$$Yr = 49.94 + 1.576X_p$$

[3] X_n stands for arc tangent percent nitrogen.

Total sum of squares = $575{,}754.2 - (6{,}182.6)^2/73 = 52{,}130.3$

Reduction in sum of squares due to regression = $1.576(7{,}583.9) = 11{,}952.2$

Reduced sum of squares = $52{,}130.3 - 11{,}952.2 = 40{,}178.1$

Reduced variance = $40{,}178.1/71 = 565.9$

F value for significance of regression = $11{,}952.2/565.9 = 21.12$

The regression of relative yield on arc tangent percent phosphorus is significant at the 1-percent level.

Potassium

$$n = 43;\, SX = 2{,}625.5;\, SYr = 4{,}008.3;\, S(XYr) = 246{,}698.9;$$

$$SX^2 = 162{,}530.8;\, SYr^2 = 382{,}220.9$$

$$B = \frac{246{,}698.9 - (2{,}625.5)(4{,}008.3)/43}{162{,}530.8 - (2{,}625.5)^2/43} = 0.882$$

$$A = 4{,}008.3/43 - 0.882(2{,}625.5/43) = 39.37$$

$$Yr = 39.37 + 0.882X_k$$

Total sum of squares = $382{,}220.9 - (4{,}008.3)^2/43 = 8{,}582.1$

Reduction in sum of squares due to regression = $0.882(1{,}959.6) = 1{,}728.4$

Reduced sum of squares = $8{,}582.1 - 1{,}728.4 = 6{,}853.7$

Reduced variance = $6{,}853.7/41 = 167.2$

F value for significance of regression = $1{,}728.4/167.2 = 10.34$

The regression of relative yield on arc tangent percent potassium is significant at the 1-percent level.

Summation

Summarizing, the formulas obtained relating yield and nutrient composition of the sorghum are:

Nitrogen	$Yr = 4.56 + 1.617\ X_n$
Phosphorus	$Yr = 49.94 + 1.576\ X_p$
Potassium	$Yr = 39.37 + 0.882_k$

The regressions of relative yield on arc tangent percent nutrients are all highly significant.

To test the adaptability of the arc tangent equation to data other than those for sorghum, experimental data were taken from the literature for corn, tomatoes, and barley. These experiments were performed under a wide variation of environmental and experimental conditions. The data used consisted of yields and the nutrient composition of the plants. The arc-tangent formula was applied to the data of Krantz and Chandler (10)[4] for nitrogen on corn yields, Hoagland and Martin (7)[5] for potassium, Kraus and Kraybill (11) for nitrogen on tomato yields, and Macy (12)[6] for nitrogen on barley-grain yields. The regression coefficients were significant at the 1-percent level for all the data except the barley, in which case it was significant at the 5-percent level. The equations obtained were as follows:

Corn	$Yr = -232.35 + 4.72\ X_n$
Tomatoes	$Yr = 18.44 + 1.52\ X_n$
	$Yr = 51.04 + 0.59\ X_{k_20}$
Barley	$Yr = 48.64 + 2.27\ X_n$

DISCUSSION

In selecting a certain type of equation or mathematical relationship to explain the relation between the nutrient contents of the plant tissues and the yield of a crop consideration should be given not only to the accuracy of the fit, or the precision of the equation to explain the desired relation, but also to the nature of the mathematical relationship and to the possible physical significance of the constants or parameters of that equation. The use of the straight-line equation to express the relationship would require the calculation of two constants, one of which would correspond to the yield obtained with zero content of the nutrient and the other would be the average constant increase in yield per unit increase in percentage nutrient content. This type of relationship requires the utilization of the concept that the relationship between the nutrient content and the yield content is one that holds from minus infinity to plus infinity.

This contrasts with the actual physical fact that nutrient contents cannot have a wider range than from 0 to 100 percent and yields cannot be negative. The use of the second-degree equation, the so-called parabola of the second order, requires the calculation of three constants, two of which are opposite in sign and none of which can be related easily to the physiological behavior of the plant. The only justification for the use of this equation, in case it is precise enough, would be the precision in fit. Even if the suitable equation were adapted to certain experimental data, it would be very difficult to make practical use of this equation, as its utilization would probably require the performance of a great many experiments before there would be the certainty that these constants would hold under different agricultural conditions.

[4] Table 2, Field No. C. F. 51 column and table 4, "June 22" column.

[5] Table 2, "No K pots" column.

[6] Table 3.

Through the change accomplished by converting percentages into angles where the arc tangent relationship is used, the apparent curvilinear relation between plant yield and composition is transformed into a straight-

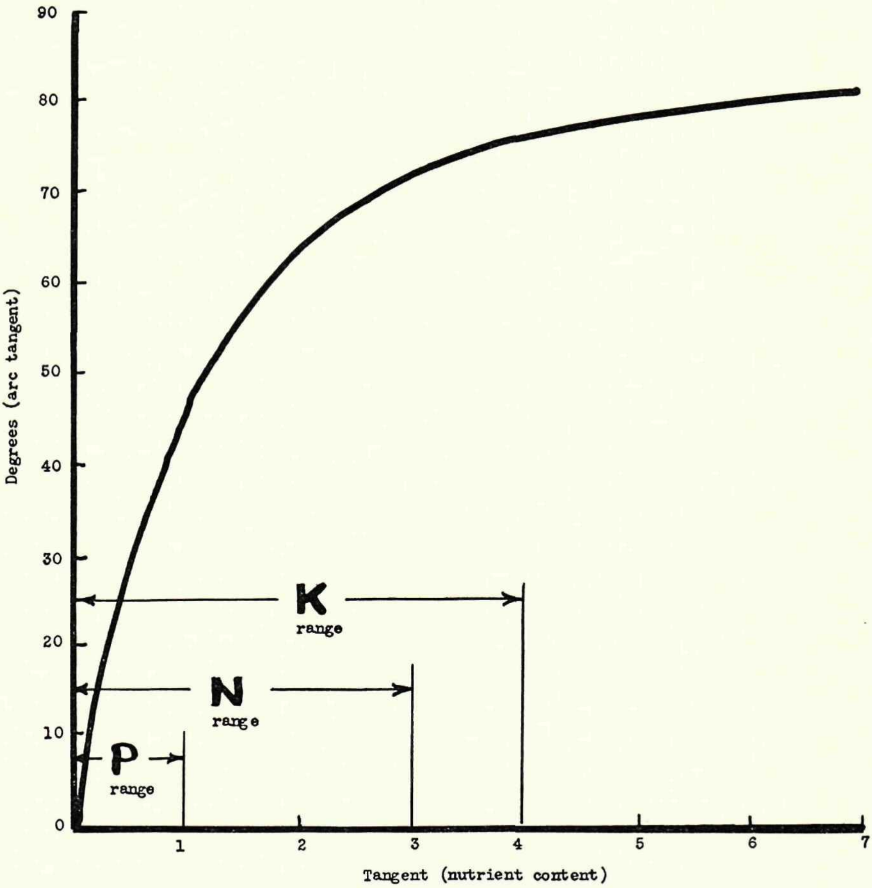

FIG. 1.— Variation of the arc tangent with the tangent and the nutrient ranges found in sugarcane leaf samples.

line relationship. Furthermore, there are limits to the values within which this relation holds, since the angle must be at all times between 0 and 90°. Therefore, this also limits the values of the relative yields which, of course, can vary only from 0 to 100 percent. The values of the tangent change slowly in the interval from 0 to 4, whereas the values of the corresponding angles vary from 0 to approximately 80 (see fig. 1). As the majority of the economic crops give nutrient-content values in the 0- to 4-percent range, and relative-yield values in the 0- to 80-percent range, we have in the arc-tangent concept a sensitive means of relating small increases in nutrient content with larger yield increments.

Furthermore, for the practical utilization of this equation, that is, in order to make decisions as to which one of a number of possible nutrients should be applied in greater amounts to a soil, the composition of the plant should be sufficient to indicate the fertility status of the growth medium. The use for this purpose of techniques which depend on yields, like the Mitscherlich's method of soil-testing, require observations under two or more fertilizer conditions so as to eliminate the effect of those environmental factors other than that of the nutrient under study. With respect to the mathematical calculations necessary to fit and interpret the arc-tangent equation they are almost as simple as the equation of the straight line and considerably simpler than the equation of the second degree, since the equation becomes a linear one when the percentage content is transformed into the angle.

The equation $Yr = A + BX$ seems to hold for each one of the three major fertilizer elements: Nitrogen, phosphorus, and potassium. Moreover, the fitted equation holds for different soils and fertilizer applications, at different sand-to-soil ratios, and when a crop is grown in different seasons of the year. Furthermore, the material analyzed was representative of whole sorghum plants at the end of their vegetative growth, that is, when they were about to head. At this stage of growth it would be too late in most instances to utilize this information in making decisions as to the quantity and quality of fertilizers required by that crop. Other data available at present, however, tend to indicate that this same type of equation holds for other crops even better when the composition of the plant at an early stage of the growth period is used. We hope to present this information, as well as data on the effect of age of crop on nutrient content, in a later paper. For perennial crops, or crops with a long period of growth, the use of this technique would be specially valuable.

SUMMARY

1. Although much work has been done on the relation between the composition of a plant and its yield, very little specific information exists about the quantitative relationship between plant yields and plant composition.

2. Using data from hegari sorghum grown at various nutrient levels on leading Puerto Rican soils, a general mathematical equation was established relating the sorghum yield to its nutrient composition. The equation is:

$$Yr = A + B \; arc \; tan \; percent \; Nu$$

where Yr is the relative yield of the crop, A and B are constants, and *arc tan percent Nu* is the arc whose tangent is the percentage of the respective nutrient in the plant on a dry-matter basis. Substituting X for arc tangent percent Nu, it can be expressed in the form of the linear equation

$$Yr = A + BX.$$

3. The arc tangent equation was developed from 50 experiments with different soil-to-sand ratios and different growing seasons, using samples of 28 different soil series. The equations obtained relating yield to the nutrient contents of hegari sorghum were:

For nitrogen, $Yr = 4.56 + 1.617\, X_n$
For phosphorus, $Yr = 49.94 + 1.576\, X_p$
For potassium, $Yr = 39.37 + 0.882\, X_k$.

The explanation of yield differences offered by these equations were highly satisfactory as indicated by the fact that the regressions were significant at the 1-percent point.

4. The use of the arc tangent equation to relate yield and nutrient composition was shown to have many practical advantages over other possible mathematical equations which might be used. The primary value of the use of the arc tangent equation is that it makes use of plant composition and does not depend on yields. Only one value obtained for any particular nutrient content is needed to make an estimate as to the sufficiency of the particular nutrient in the growth medium. Other methods usually require field experiments with samples taken at various fertility levels.

5. The arc tangent equation was applied to the data of Krantz and Chandler for corn, Hoagland and Martin and Kraus and Kraybill for tomatoes, and Macy for barley. It gave a highly satisfactory explanation of the variations in yield, as witnessed by the fact that the regressions were statistically significant at the 1-percent point in all but the last case, in which it was significant at the 5-percent point.

RESUMEN

1. Aunque se ha hecho mucho trabajo con respecto a la relación entre la composición química de la planta y el rendimiento de la misma, al presente existe muy poca información en cuanto a la relación cuantitativa entre estos dos factores.

2. Usando datos obtenidos con sorgo hegari crecido en varios niveles de abonamiento en los principales suelos de Puerto Rico, se ha establecido una ecuación matemática general que relaciona el rendimiento de sorgo con su composición química. Esta ecuación es,

$$Yr = A + B \textit{ arco tangente } N_u,$$

en donde Yr es el rendimiento relativo de la cosecha, A y B son constantes y *arco tangente* N_u es el arco cuya tangente es el por ciento del elemento nutritivo respectivo en la planta a base seca. Substituyendo X por arco tangente N_u la ecuación matemática mencionada arriba puede transformarse en la ecuación de la línea recta

$$Yr = A + BX.$$

3. La ecuación del arco tangente fué desarrollada de 50 experimentos llevados a cabo con proporciones distintas de suelo y arena en diferentes épocas de crecimiento y usando suelos de 28 series de Puerto Rico. Las ecuaciones que relacionan el rendimiento con el contenido de elementos nutritivos por el sorgo hegari fueron:

para nitrógeno: $Yr = 4.56 + 1.617\ X_n$;
para fósforo: $Yr = 49.94 + 1.576\ X_p$;
y para potasio: $Yr = 39.27 + 0.882\ X_k$.

Las explicaciones de las diferencias en rendimiento ofrecidas por estas ecuaciones fueron satisfactorias en alto grado según lo indica el hecho de que las regresiones fueron significativas en el punto del 1 por ciento.

4. El uso de la ecuación del arco tangente para relacionar el rendimiento con la composición química de la planta ha demostrado tener un número de ventajas prácticas sobre otras ecuaciones matemáticas que podrían usarse para este propósito. El valor principal del uso de la ecuación del arco tangente es que hace uso de la composición química de la planta y no depende de rendimientos. Para hacer un estimado en cuanto a la suficiencia o deficiencia de un elemento nutritivo en particular en el medio de crecimiento hace falta solamente conocer el contenido de este elemento nutritivo por la planta. Los otros métodos que pueden usarse para este propósito requieren rendimientos obtenidos bajo varios niveles de fertilidad.

5. La ecuación del arco tangente fué ajustada a los datos experimentales de Krantz y Chandler con maíz, Hoagland y Martin y Kraus y Kraybill con tomates, y Macy con cebada. La ecuación explicó satisfactoriamente en alto grado las variaciones en rendimiento obtenidas en estos experimentos como lo demuestra el hecho de que las regresiones fueron significativas estadísticamente en el punto del 5 por ciento en el último caso y en el punto del 1 por ciento en los demás.

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