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A New Approach to the Problem of Adequate Fertilization of a Crop

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INTRODUCTION

The determination of the optimum fertilizer application to make to a soil for maximum profit of the grower has been a problem that has puzzled agriculturists for a long time. The solution of this problem has been attempted from many different angles, including soil and plant analyses and the performance of laboratory, greenhouse, and field tests. The author has been greatly interested in this problem and has worked on it for the past three decades. This paper presents what he considers to be a new approach to the solution of the problem.

THE PROBLEM

The purpose of making a fertilizer application to a soil is to complement the capacity of the soil to provide the crop growing in it with the nutrients needed by the crop for growth and development. As regards the yield of the crop, *per se*, the optimum fertilizer application will be that which will enable the crop to produce a maximum yield of product of the desired quality. As regards the grower, however, the optimum fertilizer application is that which will enable him to obtain a maximum return from his investment in the business. Knowledge as to how the different concentrations of the various nutrients in the growth medium influence the yield of the crop is essential, but insufficient to solve the latter of the two problems.

THE SUGGESTED APPROACH

To simplify the presentation, consideration will be given first to the problem of determining the optimum application of one nutrient only. Later on the more complex problem of determining simultaneously the optimum application of several nutrients will be dealt with.

It is hereby suggested that the estimation of the optimum amount of a

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fertilizer material to apply to the soil for the commercial production of a crop be made according to the following procedure:

1. Take a representative sample of the appropriate tissue of the plants growing in the field that are to receive the fertilizer application. This sample should be taken about when the fertilizer is normally applied to the crop.

2. Analyze the sample to determine the percentage content of the particular nutrient in it.

3. By making use of a previously determined relation between plant-tissue analysis and available nutrient content of the growth medium, estimate the available amount of the nutrient in the soil. How to determine this relation is discussed later.

4. Estimate the quantity of the respective nutrient that should be in available form in the soil to enable the grower to make a maximum profit. This is done by making use of another previously determined relation indicating how the net profit of the grower is influenced by: 1, The quantity of fertilizer material to apply to the soil; 2, the cost of one unit of the fertilizer material, including its cost of application; 3, the cost of harvesting, transporting, and marketing one unit of the crop; 4, the selling price of the crop; and 5, the expected adequacy of growing conditions. The determination of this other relation will also be discussed later.

5. By then subtracting the quantity of the fertilizer material already available in the soil from the quantity needed for optimum economic benefit, estimate the total amount of the fertilizer material that should be applied to the soil.

The evidence on which this procedure rests is discussed below.

ESTIMATION OF THE AVAILABLE NUTRIENT CONTENT OF A SOIL ON THE BASIS OF THE PLANT-TISSUE CONTENT OF THE RESPECTIVE NUTRIENT

It is hereby submitted that at a certain stage of the yield cycle of a plant, the available quantity of a nutrient in the growth medium and the quantity of the same nutrient in certain tissues of plants growing thereon are definitely related. The basis for this claim is as follows:

1, There is abundant evidence that the yield of a crop is related in a definite way to the available concentration of nutrients in the growth medium. This will be discussed later in more detail.

2, There is also evidence that the yield of the crop is related in a definite way to the nutrient contents of the plant tissue at a certain stage of the yield cycle. This also will be discussed later.

3, Since yield is a factor common to both these relations, it is obvious that there must be a definite relation between the content of a nutrient by the plant tissue at a certain stage of its yield cycle and the quantity of the nutrient available in the growth medium. It is possible, therefore, to estimate the available nutrient content of a soil from the analysis of the plant tissue of the respective plants growing thereon.

THE RELATION BETWEEN YIELD AND THE AVAILABLE NUTRIENT CONTENT OF THE GROWTH MEDIUM

There is a range of concentrations of any given essential nutrient within which plant growth takes place. With very low concentrations this process proceeds at a reduced rate. With higher concentrations this rate increases, until a maximum is attained with an optimum concentration of the nutrient. Beyond this optimum nutrient level harmful effects set in and the rate of growth is drastically reduced with increasing concentrations of the nutrient. Russell (*17*)[2], among many other research workers, has discussed the way in which the concentration of a certain nutrient influences plant growth. There has been considerable discussion with regard to the mathematical model that best describes the relation between nutrient concentration and crop yield, but there is no doubt that there is a definite relation that can be explained fairly well within commercial limits of fertilizer applications by several simple mathematical relations. Among others, the following relatively simple mathematical models have been proposed for use in estimating available nutrient contents from given crop yields and vice versa: 1, The algebraic equation of the second degree proposed by Pfeiffer (*12*): $Y = A + BX + CX^2$; 2, the Mitscherlich or Spillman exponential yield curve (*4, 5, 7, 10, 12, 14, 15, 16, 19, 20*): $Y = K(1 - R^x)$; 3, the Robertson formula (*12*): $\log Y - \log (A - Y) = X - X_1$; 4, the Maskell resistance formula (*1, 11, 13, 17*): $\frac{1}{Y} = \frac{A}{N} + K$; and 5, the law of mass action (*4, 6, 18*): $Y = A(B^x)$. In all these equations Y represents crop yield and X represents the nutrient concentration in the growth medium.

With the exception of the Robertson formula, none of the graphs of these mathematical functions has any point of inflexion as the nutrient concentration increases from low to medium values, which, as Russell (*17*) has stated "accurate working often reveals" to be present. Again, none of these equations provides for the fact that excessively large concentrations of any nutrient in the growth medium injure plant growth and thereby reduce yields. To provide for this latter possibility Mitscherlich (*17*) proposed a modification to his equation regarding which Russell (*17*) stated that it "is a useful approximation to the case where one factor only varies, all others being constant", although it does not provide for the point of inflexion. In this connection Bondorff (*17*) suggested the equation

[2] Italic numbers in parentheses refer to Literature Cited, pp. 252–3.

$Y = CX^m - KX^n + A$, which provides for the harmful effect of excessively large concentrations of the nutrient and, when m is greater than 1, for the point of inflexion also.

The evidence referred to points to the existence of a definite relationship between the available nutrient level in the growth medium and the yield of the crop that grows thereon. Not only that, but it also shows that this relation may be expressed satisfactorily by relatively simple mathematical models.

THE RELATION BETWEEN RELATIVE YIELD AND NUTRIENT CONTENT OF THE PLANT TISSUE

Capó (*8*) reported, and Capó and Samuels (*9*) presented evidence on the existence of a relationship between the plant-tissue contents of some plants and the yields of the respective crops. The mathematical model suggested to represent this relation was the arc-tangent equation $Y = A + B \tan^{-1} X$, where Y is the relative yield of the crop and X is the content of the nutrient in percentage of the dry weight of the plant tissue. With this equation it was possible to explain with a high precision variations in Hegari sorghum yields due to variations in content of nitrogen, phosphorus, and potassium in the sorghum tissue. This equation has been found to hold also for sugarcane, coffee, grasses, tomatoes, and other crops (*2, 3, 9, 22*).

RELATION BETWEEN AVAILABLE NUTRIENT CONTENT OF THE GROWTH MEDIUM AND THE NUTRIENT CONTENT OF THE PLANT TISSUE

There being a definite relation between crop yield and available nutrient content of the soil on the one hand and also between crop yield and plant-tissue nutrient content on the other, it is obvious that there is a definite relation between plant-tissue or leaf-nutrient content and the respective available nutrient content of the soil. It is possible, therefore, to estimate the nutrient-availability status of the soil by analyzing a sample of the plant tissue taken at the proper time in the growth period. The proposed approach, therefore, contemplates estimating the available nutrient content of the soil on the basis of the nutrient content of a sample of plant tissue.

ESTIMATION OF THE OPTIMUM AMOUNT OF FERTILIZER TO APPLY TO THE SOIL FOR MAXIMUM ECONOMIC RETURN TO THE GROWER

It is possible to use one of the above-mentioned fertilizer-yield equations to estimate the most profitable amount of fertilizer to apply to the soil. This has been done by many workers. Prescott (*17*) did this with superphosphate for wheat. The author has done it with simultaneous variations of nitrogen, phosphorus, and potassium for sugarcane. Spillman (*19*) and Taboada (*21*) have described methods applicable to this type of calculation.

REMARKS ON THE PROPOSED APPROACH TO THE SOLUTION OF THE PROBLEM

The utilization of the equations mentioned above for the determination of the available nutrient contents of the soil by fitting the respective equations to yield and chemical data obtained with different fertilizer levels requires the performance of a yield test. The respective fertilizer-yield equation is fitted to the yield data of the test, and one of the constants of the equation represents the quantity of the nutrient available to the crop. Thus, for example, in using the Mitscherlich equation for this purpose, the equation to be fitted would be of the form

$$Y = K(1 - R^{X_s + X_F}),$$

where the X is now expressed as $X_s + X_F$, where X_s represents the available nutrient in the soil which has received no fertilizer, and X_F represents the nutrient supplied by the fertilizer. This equation has the constants K, R and X_s which are evaluated in the process of fitting the equation to the yield data. To make these evaluations the yield test must include at least three different nutrient levels, so that there will be at least three yield observations to which to fit the equation. The fertilizer levels to be included in these tests must represent as wide a range of concentrations as possible, that is, from very low to very high, even if some are harmful, concentrations.

Since this procedure requires the performance of a yield test to evaluate the available nutrient content of every field or area to be fertilized on the basis of the information thus obtained, great emphasis is generally given to the use of simple equations which require only simple tests, in order to reduce the work required to perform the test and to fit the equation. Mitscherlich once claimed that the value of R was constant for a nutrient and that it applied to all crops under all climatic and soil conditions. On the basis of this assumption, the Mitscherlich method of soil testing is based on the use of an indicator crop, it being assumed that what is available to the indicator crop is available to other crops. This is not so, there being data available indicative of the differential response of different crops to fertilizer applications in the same soil and, even of different varieties of the same crop growing in the same soil.

Under these conditions, the usefulness of indicator plants is reduced, and there is need to carry out the tests with the commercial crop to be grown. But this is again a practical impossibility, since: 1, Field tests are exceedingly costly if they are to include enough replications to yield reliable data, and 2, the information on nutrient-status availability is obtained only after performing and interpreting the results of the field test. This information would be useful only for future crops and not for the one underway, and the information is possibly insufficiently precise for future crops.

The advantage of the approach suggested in this paper is that neither field tests nor indicator plants are required on a routine basis, the calculations being based on the analysis of the plant tissue taken at about the time that the fertilizer is normally applied to the crop underway. The tests to be performed are needed only initially to calibrate the method with the crop, in a soil and under climatic conditions similar to those under which the commercial production is expected to be undertaken. Because of this a more precise, even if more complicated equation, can be used as an approximation to the relation between available nutrient content and crop yield.

As to soil chemical tests it may be said that there does not seem to be any method of chemical analysis capable of determining with sufficient precision the available soil content of the different nutrients required by any given crop. The maximum precision claimed for the methods in use is one in which the order of availability is expressed in terms of a relatively few classifications of availability, such as very low, low, fair, large, and very large. There is no possibility of using this type of information in calculating optimum economic applications of fertilizers.

HOW TO OBTAIN THE RELATION BETWEEN THE NUTRIENT CONTENT OF THE PLANT TISSUE AND THE AVAILABLE NUTRIENT CONTENT OF THE SOIL[3]

The arc-tangent equation previously mentioned relates plant-tissue content with relative yields as follows:

$$Y_r = A + B \tan^{-1} X_p ,$$

where Y_r is the relative yield, X_p is the nutrient content in the plant tissue, and A and B are constants.

[3] This presentation is based on the assumption that the preferred fertilizer-yield equation to be used is the Mitscherlich-type equation. The same procedure is applicable should it be desired to use any other type of fertilizer-yield equation. The cubic equation:

$$Y = A + BX^2 + CX^3,$$

which is a form of the Borndorff equation, is probably more precise for this purpose than the Mitscherlich equation since, as indicated earlier, it provides for the point of inflexion and for the reduction in yield due to the harmful effect of excessive concentrations of a nutrient. This equation requires the performance of calibration yield tests with at least four different fertilizer levels. With four or more fertilizer levels, fitting of this cubic equation is easier and less time-consuming than fitting the Mitscherlich equation.

In the Mitscherlich equation the yield of the crop is related to the available nutrient amount in the soil as follows:

$$Y = K(1 - R^{X_s}),$$

where Y is the yield of the crop, X_s is the available nutrient in the soil and K and R are constants, K representing the maximum yield. The relative yield is, therefore,

$$\frac{Y}{K}, \text{ which means that}$$

$$\frac{Y}{K} = Y_r = 1 - R^{X_s}$$

Since from the arc-tangent equation,

$$Y_r = A + B \tan^{-1} X_p,$$

and from the Mitscherlich equation,

$$Y_r = 1 - R^{X_s},$$

it follows that

$$1 - R^{X_s} = A + B \tan^{-1} X_p;$$

that is,

$$X_s = \frac{\log (1 - A - B \tan^{-1} X_p)}{\log R}.$$

It is mathematically possible to develop similar relations between nutrient content of the plant tissue and available nutrient content of the soil by using any other one of the fertilizer-yield equations previously discussed. In some of these other cases, however, the mathematical relation expressing relative yield is more complex than in the case of the Mitscherlich equation, and it is doubtful whether in commercial practice it might be more advisable to use a more complex, even if more precise relation, in preference to the one suggested.

HOW TO ESTIMATE THE QUANTITY OF THE NUTRIENT THAT SHOULD BE PRESENT IN THE SOIL TO ENSURE THE GROWER A MAXIMUM PROFIT

Spillman (*19*) suggested the use of the following relation to estimate the quantity of a fertilizer which should be applied to the soil for a maximum profit of the grower:

$$P = VY - QX - C,$$

where P is the profit per unit of area; V is the difference between the selling price and the cost of harvesting, transporting, and marketing 1 unit of the crop; Y is the yield of the crop per unit of area; Q is the cost of 1 unit of the fertilizer, including the cost of its application; X is the quantity of fertilizer applied per unit of area, and C is a constant representing all other production costs.

When the Mitscherlich equation is used as the fertilizer-yield equation, the equation becomes

$$P = VK(1 - R^x) - QX - C.$$

The quantity of the nutrient that need be present in the soil for a maximum profit of the grower may be obtained by solving for X the equation resulting from equating with zero the derivative of P with respect to X.

This gives $X = \dfrac{\log H + \log Q - \log V}{\log R}$

where $H = 0.43429/K(-\log R)$.

DISCUSSION

The approach suggested is also applicable to cases where more than one fertilizer element is considered. In this case, however, the fertilizer-yield equation is more complex, that of the Mitscherlich type being

$$Y = K(1 - R_1^{X_1})(1 - R_2^{X_2}) \dots (1 - R_N^{X_N}),$$

while that of the cubic equation type might be

$$Y = A(1 + B_1X_1^2 + C_1X_1^3)(1 + B_2X_2^2 + C_2X_2^3) \cdots (1 + B_NX_N^2 + C_NX_N^3).$$

The arc-tangent equation need not be modified since the information available seems to indicate that the relation between the nutrient content of the leaf and the relative yield is not appreciably affected by the levels of the other growth factors, when these are not present at extreme levels, and this is not the normal case in agricultural soils.

The above-mentioned Mitscherlich equation for the simultaneous variation of several nutrients or growth factors has been criticized because it does not provide for the possible interaction of the growth factors involved. In fact, the author has shown that, when the constants of the equation are estimated from the results of experiments in which the maximum fertilizer levels used are kept at relatively low values, it may be possible to overestimate the yield when making calculations for conditions of heavy fertilization. If, however, the constants are estimated from experiments in which the maximum fertilizer levels used are high, the error of estimate of yields expected with lower levels of fertilization, if of any significance whatsoever, would tend to be conservative and lead to no serious errors. This is probably true with respect to the use of any other of the above-mentioned fertilizer-yield equations which do not provide for the possible interaction of the different growth factors.

It would, of course, be preferable to make use in this connection of a relation which takes care of these factor interactions. This type of equation, however, is not only lacking at present, but is almost, if not altogether impossible to derive, fit, and use in practice. The situation, therefore, is one in which the decision has to be taken whether to continue the hitherto used relatively inefficient methods of estimating the fertilizer requirements of agricultural soils for the production of economic crops, or to make use of improved procedures already available, such as that made possible by the new approach suggested here for the solution of this problem. It seems to the author that the latter is by far the sounder of the two procedures.

SUMMARY

At present there is available no method of soil analysis which provides the information required to estimate with enough precision the quantities of the various nutrients that must be added to a soil to ensure maximum profit to the farmer. Although the results of certain field experiments make possible the estimation of the optimum rate of fertilization required for the commercial production of a crop, it is necessary to wait until the experiment is harvested, and its results are interpreted to carry out this estimation. The information provided by experiments performed with indicator plants prior to the time of fertilization of the commercial crops is as unreliable as that provided by the chemical analysis of the soil.

The author has found a mathematical equation that can be used with a high degree of precision to represent the relationship between the content of a nutrient by the plant tissue at a certain stage of its growth cycle and the relative yield of the plant of which the tissue is analyzed. By using this mathematical equation jointly with a relation between the available nutrient content of the soil and the yield of the crop grown in that soil, it is possible to estimate also, with a high degree of precision, the quantities of the different fertilizer elements that should be added to the commercial crop so that the farmer may obtain a maximum profit.

The relationship between the nutrient content of the plant tissue and the yield of the crop is:

$$Y = A + B \tan^{-1} X,$$

in which A and B are constants, X is the percentage nutrient content of the tissue on a dry basis, and Y is the relative yield of the crop.

As an approximation to the relationship between the available nutrient content of the soil and the yield of the crop the following cubic equation is suggested among others:

$$Y = A + BX^2 + CX^3.$$

RESUMEN

Al presente no existen métodos de análisis de suelos que ofrezcan información que permita estimar con precisión las cantidades de los distintos nutrientes que se requiere añadir a un suelo para una ganancia máxima por el agricultor. Aunque los resultados de experimentos de campo permiten estimar estos requisitos óptimos de abonamiento para la producción comercial de una cosecha, es necesario aguardar a cosechar e interpretar los resultados de dichos experimentos para disponer de esta información. Los datos suministrados por experimentos realizados con plantas indicadoras con antelación a la época de abonamiento de las cosechas comerciales adolecen de la misma falta de precisión que la información suministrada por los análisis químicos de los suelos.

El autor ha descubierto la existencia de una relación matemática que puede utilizarse para representar con un alto grado de precisión la relación entre el contenido de un nutriente por el tejido vegetal en cierta etapa del ciclo de producción y el rendimiento de la planta cuyo tejido se analiza. Utilizando esta relación conjuntamente con la relación entre el contenido asimilable de nutrientes por el suelo y el rendimiento del cultivo sembrado en dicho suelo, es posible estimar con un alto grado de precisión las cantidades de nutrientes que deben añadirse a un campo comercial para que el agricultor obtenga un beneficio económico máximo.

La relación entre el contenido de un nutriente en el tejido vegetal y el rendimiento de la cosecha es:

$$Y = A + B \tan^{-1} X,$$

en donde A y B son constantes, X es el contenido en por ciento, a base seca, del nutriente en el tejido vegetal, y Y es el rendimiento relativo de la cosecha.

Como aproximación a la relación entre el contenido asimilable de un nutriente por el suelo y el rendimiento de la cosecha se menciona entre otras la ecuación algebraica del tercer grado:

$$Y = A + BX^2 + CX^3.$$

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