THE JOURNAL OF AGRICULTURE
OF THE UNIVERSITY OF PUERTO RICO

Issued quarterly by the Agricultural Experiment Station of the University of Puerto Rico, for the publication of articles by members of its personnel, or others, dealing with any of the more technical aspects of scientific agriculture in Puerto Rico or the Caribbean Area.

Vol. LI | April 1967 | No. 2

Additional Evidence on the Applicability of the New Fertilizer-Yield Relation

B. G. Capó[1]

INTRODUCTION

The nature of the fertilizer-yield relation has been a controversial subject, widely divergent concepts having been suggested by different workers as approximations to this relation. Among the concepts advanced the following may be mentioned:

1. A straight-line relation between certain limits. Liebig's Law of the Minimum (*7*)[2] assumes a constant uniform increase in Y with increases in X.

2. A smooth concave-downwards relation, the crop yield approaching a maximum value as an asymptote with unlimited increases in the concentration of the corresponding nutrient. This concept has been the basis of:

A, Mitscherlich's (*6*) or Spillman's (*9*) logarithmic relation.

B, Balmukand's (*1*) suggestion of the applicability of the Maskell's resistance formula to this purpose.

C, The Law of Mass Action, as suggested by Sapehin (*8*).

3. A concave-downwards curve, passing through a maximum value at finite values of X. This concept is the basis of:

A. The parabola, as suggested by Pfeiffer (*5*).

B. The Mitscherlich's modified equation (*7*).

4. A curve the concavity of which changes from concave upwards to concave downwards at a certain point of inflexion, approaching beyond this point a maximum value as an asymptote. This concept is the basis of Robertson's (*5*) formula.

5. A curve the concavity of which changes from upwards to downwards as X increases below a certain value, and then changes again from downwards to upwards beyond this value. This concept is the basis of the new fertilizer-yield equation suggested by the author in a previous article (*3*).

The variously proposed concepts, of course, have been advanced by the various workers as results of the studies of the nutrient-crop yield data

[1] Technical Consultant, Agricultural Experiment Station, University of Puerto Rico, Río Piedras, P.R.

[2] Italic numbers in parentheses refer to Literature Cited pp. 119–20.

available to the particular worker who made the respective suggestion. Certainly there are yield data which seem to be satisfactorily fitted by one or another of the relations mentioned above. All the above relations, however, with the exception of the new fertilizer-yield equation suggested by the author, are too limited in their applicability, adapting themselves to certain types of data and not being adapted at all to other types of yield data. The new fertilizer-yield equation suggested by the author is, on the contrary, applicable to most types of data of this kind, as far as the author knows.

In this article, additional data are presented to demonstrate the closeness of the fit obtained when using the new fertilizer-yield equation to explain fertilizer-yield data of sugarcane and corn experiments.

The equation suggested by the author to represent the fertilizer-yield relation is

$$Y = \frac{A}{1 + B(X - C)^2},$$

where X is the quantity of fertilizer applied to the soil and Y is the yield of the crop grown thereon. In this equation A, B, and C are parameters. A represents the maximum yield obtainable in the given field with the optimum fertilizer application, C, under the prevalent climatic and other environmental conditions. B may be assumed to be an index of the variability of the crop yield as the quantity of the respective fertilizer material applied differs from the optimum application, C.

The equation is symmetrical, being concave upwards for low or high values of X, and concave downwards for values of X near C.

EXPERIMENTAL DATA

At the Agricultural Experiment Station of the University of Puerto Rico a number of experiments with various crops and fertilizer materials were planned to gather data which might be useful, among other purposes, to better evaluate the nature of the fertilizer-yield relation. These experiments were planned to be carried out with many crops and fertilizer materials in many soils representative of the various crop-producing areas of Puerto Rico.

As of the early Spring of 1964, 20 experiments had been performed with corn and 14 with sugarcane, at 5 locations on the Island. In each of these 34 experiments[3], only 1 nitrogen-, phosphorus-, potassium-, calcium-, or

[3] These experiments were designed by Dr. J. A. Bonnet and Messrs. A. Riera and J. Juárez, with the advice of the author. Messrs. Riera and Juárez installed these experiments and were mainly responsible for their harvesting and collection of samples. Many other members of the staff and private cooperators participated in this work.

magnesium-bearing fertilizer material was varied, while the other fertilizer materials were applied uniformly in quantities considered to be adequate for satisfactory crop yields. Nine levels of the fertilizer material under study were used in the corresponding experiment, the applications varying from zero to quantities considered *a priori* to be sufficient to be harmful to the yield of the crop used.

Tables 1 to 10 (introduced later) provide the following data with regard to each experiment: Crop and variety, geographical location, soil type of the experimental field, planting and harvesting dates, fertilizer treatment, and crop yield.

STATISTICAL CALCULATIONS

Although the above-mentioned experiments were set up as simple lattices (nine treatments, each replicated two times), the treatment mean yields were calculated as if the experiments had been set up as fully randomized block experiments. The previously mentioned fertilizer-yield equation was then fitted to the yield data of each experiment.

For the former article by the author on this subject (*3*), the equation was fitted to the data, as indicated there, by expanding the equation by Taylor's theorem, estimating in a simple way approximate values for the parameters, and estimating the corrections to these approximate values by the method of least squares. The data presented herein have been processed in a different manner, since the above-mentioned procedure did not insure the best fits when only the first partial derivatives of the expanded equation were used for the correction procedure.

For the present article, approximate values for the parameters were again estimated in a simple way, and then the corrections were estimated by iteration. Using the original approximation to C, values of B and A were tried until the sum of squares of the estimated from the observed mean yields was reduced to a minimum. With the value of B thus obtained, values of C and A were tried until the sum of squares of the above-mentioned deviations was reduced again to a minimum. When deemed necessary, a second round of corrections for B, C, and A was carried out.

All these calculations were performed by using a Japanese abacus, the columns of which were identified according to a scheme devised by the author markedly to simplify its use and reduce the time and effort needed to record the figures needed and the results of the calculations. To reduce also the labor of computation, a table was prepared indicating the values of $B(X - C)^2$ for all values of B from 1 to 70 and all values of C from 1 to 120. By placing the decimal point where required, the values of $B(X - C)^2$ for decimal fractions of a unit of both B and C were easily estimated.

The estimated treatment mean yields, the values of A, B, and C, and the sums of squares of the deviations of the estimated from the observed mean yields are also presented in the tables herein. (These estimated mean yields and sums of squares of deviations were calculated by using in all cases numbers of no more than four significant figures. Carrying or using more significant figures will probably vary some of the values presented).

TABLE 1.—*Results obtained on fitting the fertilizer-yield equation to the data of the nitrogen experiments with sugarcane variety P.R. 980*

Location	Isabela	Gurabo	Patillas
Soil type	Coto clay	Mabí clay	Viví sandy loam
Planting date	Nov. 28, 1962	Dec. 19, 1962	Nov. 3, 1962
Harvesting date	Feb. 4, 1964	Feb. 25–7, 1964	Mar. 25, 1964
Age at harvest	57.4 weeks	62 weeks	69.4 weeks

Fertilizer treatments and mean yields

Treatment No.	Units of N[1]	Y[2]	Y'[3]	Y	Y'	Y	Y'
1	0.00	96.8	89.7	58.8	68.7	71.5	83.0
2	.05	90.9	90.8	73.3	70.5	68.5	84.1
3	.10	93.0	92.0	71.6	72.4	126.0[4]	85.4
4	.15	86.6	93.0	74.9	74.2	105.5	86.6
5	.20	96.6	94.1	76.9	76.1	86.5	87.7
6	.30	85.5	96.1	84.8	79.8	95.5	90.1
7	.60	106.3	101.5	93.3	90.2	72.5	96.3
8	1.20	108.5	106.8	95.7	100.9	105.5	104.4
9	2.00	99.4	100.2	85.8	84.1	104.0	100.8

Statistics of the fitted equations

A			106.8		100.9		105.0
B			.12		.32		.13
C			1.26		1.21		1.43
Sy_ey_e[5]			238.00		172.00		2,165.00

[1] 1 unit of N = 1,000 lb. N per acre.

[2] Y = observed mean cane yield in tons of cane per acre.

[3] Y' = mean yield in tons of cane per acre estimated with the fitted equation.

[4] From the yield of 1 plot only.

[5] Sy_ey_e = sum of squares of deviations of the estimated from the observed mean yields.

RESULTS OF THE STATISTICAL CALCULATIONS

NITROGEN EXPERIMENTS

Tables 1 and 2 present the data from three sugarcane and four corn experiments with ammonium sulfate, carried on at Isabela, Gurabo, Patillas, Lajas, and Río Piedras, in which the nitrogen applications were varied from zero to 2,000 pounds of nitrogen per acre. Figure 1, A, B, and C gives an idea of the variation of the sugarcane yields with the increasing applications of ammonium sulfate, whereas figure 2, A, B, C, and D shows the variation of the corn yields with the varying applications of ammonium sulfate.

TABLE 2.—*Results obtained on fitting the fertilizer-yield equation to the data of the nitrogen experiments with corn variety Mayorbela*

Location	Isabela	Gurabo	Lajas	Río Piedras
Soil type	Coto clay	Mabí clay	Fraternidad clay	Vega Alta clay
Planting date	Mar. 28, 1963	Aug. 1, 1963	Apr. 18, 1963	July 10, 1963
Harvesting date	July 1, 1963	Nov. 12, 1963	Aug. 15, 1963	Oct. 17, 1963
Age at harvest	13.6 weeks	14.7 weeks	18.4 weeks	14.1 weeks

Fertilizer treatments and mean yields

Treatment No.	Units of N[1]	Y[2]	Y'[3]	Y	Y'	Y	Y'	Y	Y'
1	0.00	62.0	62.6	26.1	40.6	10.1	19.0	38.8	35.3
2	.05	59.3	61.6	34.3	42.0	14.4	19.2	36.9	35.3
3	.10	58.4	60.4	37.6	43.3	26.4	19.5	38.4	35.3
4	.15	57.5	59.0	51.7	44.7	25.6	19.7	34.6	35.2
5	.20	60.4	57.6	57.7	46.1	18.6	19.8	31.4	35.2
6	.30	55.1	54.4	59.3	48.9	22.7	20.2	38.1	35.0
7	.60	52.1	44.0	57.2	56.5	23.7	20.8	27.9	34.1
8	1.20	20.0	26.8	53.5	61.5	14.3	20.2	27.9	31.0
9	2.00	11.2	14.5	47.9	43.9	19.6	16.6	28.3	25.4

Statistics of the fitted equations

A			64.7		62.1		20.9		35.3
B			.70		.47		.17		.10
C			−.22		1.06		.76		.02
Sy_ey_e[4]			143.00		673.00		244.00		105.00

[1] 1 unit of N = 1,000 lb. N per acre.

[2] Y = observed mean corn yield in hundredweights of grain per acre.

[3] Y' = mean yield in hundredweights of grain per acre estimated with the fitted equation.

[4] Sy_ey_e = sum of squares of deviations of the estimated from the observed mean yields.

The graphs show how well the suggested fertilizer-yield equation fits the data of these experiments, even in the case of the corn experiment at Isabela (fig. 2, A), in which there is a change in concavity of the curve, from concave downwards to concave upwards, as the yields decrease with increasing applications of the ammonium sulfate.

The values of B in the sugarcane experiments are, in ascending order, 0.12, 0.13, and 0.32; while in the corn experiments they are 0.10, 0.17, 0.47, and 0.70.

The estimated optimum nitrogen applications, also in ascending order, are 1,210, 1,260, and 1,430 pounds of nitrogen per acre for the sugarcane yields and −220, 20, 760, and 1,060 pounds of nitrogen per acre for the corn yields.

The sum of squares of deviations of the estimated from the observed mean cane yields, (Sy_ey_e), are 172, 238, and 2,165 units, whereas the corresponding values for corn are 105, 143, 244, and 673 units.

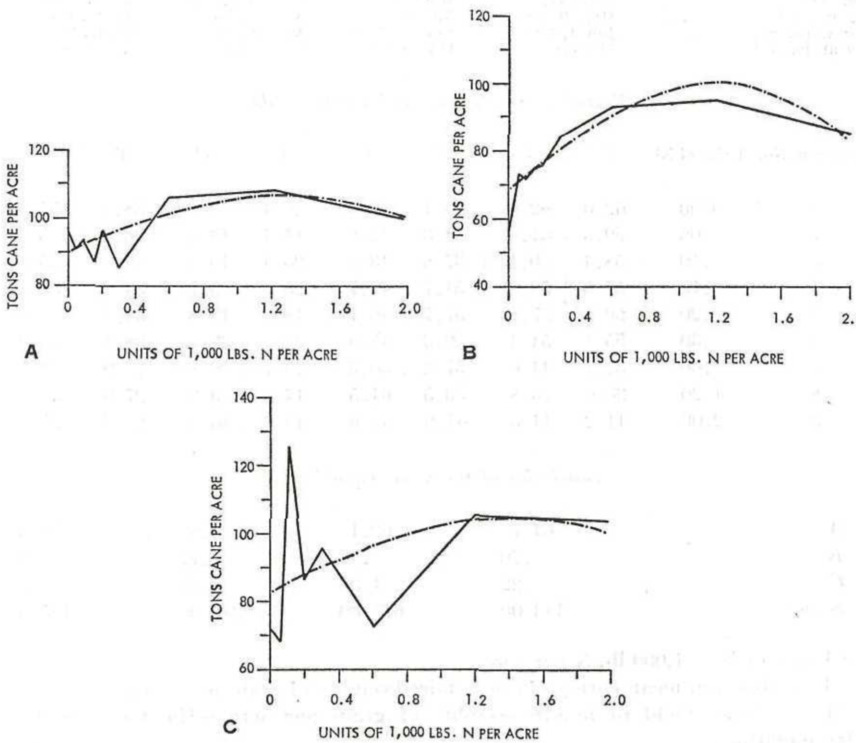

FIG. 1.—Nitrogen-sugarcane relation at: A, Isabela; B, Gurabo; C, Patillas.

PHOSPHORUS EXPERIMENTS

Tables 3 and 4 present the data from three sugarcane and four corn experiments with calcium superphosphate, in which the phosphorus applications varied from zero to 1,500 pounds of phosphorus per acre. Figure 3, A, B, and C shows how well the fertilizer-yield equation fits the yield data from the sugarcane experiments, and figure 4, A, B, C, and D gives similar information with respect to the corn experiments.

The estimated phosphorus B values for sugarcane are 0.02, 0.05, and 0.05, whereas for corn they are 0.13, 0.14, 0.23, and 0.39.

The estimated optimum phosphorus applications for cane yields are 300, 1,030, and 2,140 pounds of phosphorus per acre, while for corn they are 720, 770, 800, and 1,060 pounds of phosphorus per acre.

The sums of squares of the deviations of the estimated from the observed

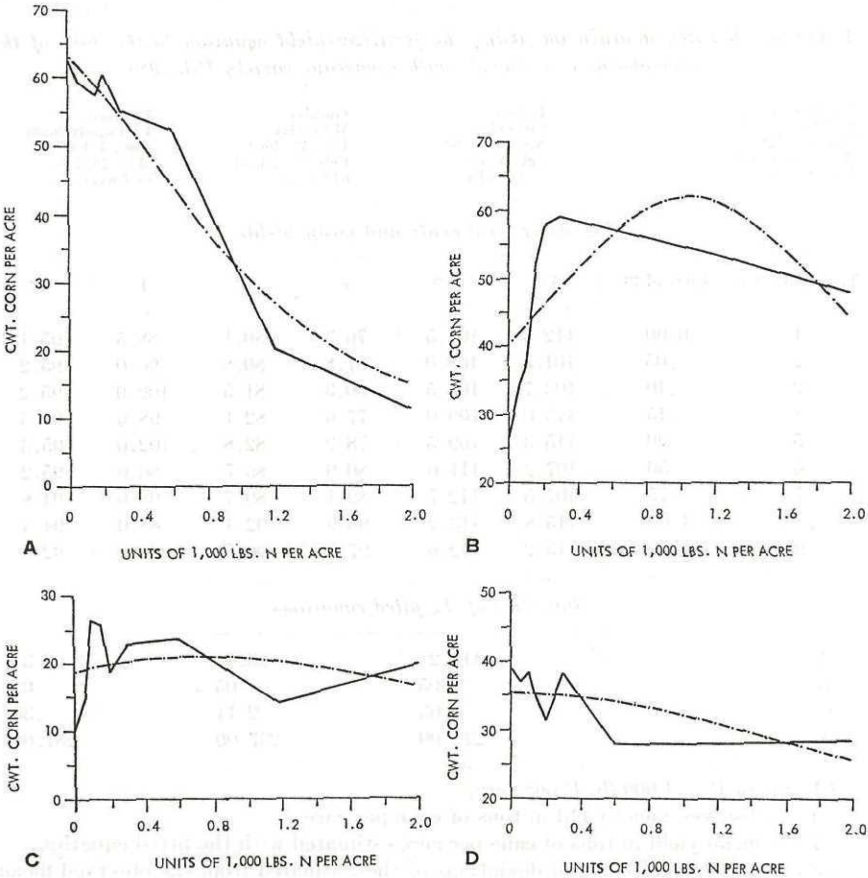

FIG. 2.—Nitrogen-corn relation at: A, Isabela; B, Gurabo; C, Lajas; D, Río Piedras.

cane yields are 212, 237, and 281 units, the corresponding values for corn being 85, 115, 188, and 151.

POTASSIUM EXPERIMENTS

Tables 5 and 6 provide the data on three sugarcane and four corn experiments with muriate of potash, in which the potassium applications varied from zero to 10,000 pounds of potassium per acre. Figure 5, A, B, and C illustrates the closeness of fit obtained with the fertilizer-yield equation when used to explain the variability of the cane yield data and figure 6, A, B, C, and D provides similar information about the corn yield data. Again, especially with reference to figures 5, B, and 6, B and D, it should be noticed how well the concept of the existence of a point of inflexion in

TABLE 3.—*Results obtained on fitting the fertilizer-yield equation to the data of the phosphorus experiments with sugarcane variety P.R. 980*

Location	Isabela	Gurabo	Patillas
Soil type	Coto clay	Mabí clay	Viví sandy loam
Planting date	Nov. 28, 1962	Dec. 19, 1962	Nov. 3, 1962
Harvesting date	Feb. 4, 1964	Feb. 25–7, 1964	Mar. 25, 1964
Age at harvest	57.4 weeks	62 weeks	69.4 weeks

Fertilizer treatments and mean yields

Treatment No.	Units of P[1]	Y[2]	Y'[3]	Y	Y'	Y	Y'
1	0.00	112.3	107.5	76.3	80.1	88.5	95.1
2	.05	101.5	108.0	87.8	80.8	96.0	95.2
2	.10	103.7	108.5	90.3	81.5	102.0	95.2
4	.15	115.0	109.0	77.6	82.1	98.0	95.3
5	.20	115.3	109.5	78.2	82.8	102.0	95.3
6	.50	107.2	111.6	80.9	86.7	86.0	95.2
7	.75	107.5	112.7	89.1	89.7	98.0	94.9
8	1.00	115.8	113.2	96.9	92.4	88.0	94.4
9	1.50	113.2	112.0	97.0	96.5	94.5	92.6

Statistics of the fitted equations

A			113.2		98.4		95.3
B			.05		.05		.02
C			1.03		2.14		.30
Sy_ey_e[4]			212.00		237.00		281.00

[1] 1 unit of P = 1,000 lb. P per acre.

[2] Y = observed mean yield in tons of cane per care.

[3] Y' = mean yield in tons of cane per acre estimated with the fitted equation.

[4] Sy_ey_e = sum of squares of deviations of the estimated from the observed mean yields.

the fertilizer-yield curve beyond the optimum fertilizer application agrees with the observed data. This agreement, to be sure, is over and in addition to the acceptable fits obtained with what might be considered, on the basis of the relatively incomplete fertilizer-yield data heretofore available, "more normal" yield data.

The estimated potassium B values for cane are 0.0004, 0.01, and 0.08, whereas the potassium B values for corn are 0.005, 0.014, 0.03, and 0.06.

The optimum estimated potassium applications as to cane yields are

1,060, 1,900, and 4,510 pounds of potassium per acre. The corresponding values as to corn yields are −140, 1,460, 1,470, and 4,600 pounds of potassium per acre.

The sums of squares of deviations of the estimated from the observed mean cane yields are 217, 526, and 676 units, while for the corn yields they are 32, 37, 115, and 271 units.

TABLE 4.—*Results obtained on fitting the fertilizer-yield equation to the data of the phosphorus experiments with corn variety Mayorbela*

Location	Isabela	Gurabo	Lajas	Río Piedras
Soil type	Coto clay	Mabí clay	Fraternidad clay	Vega Alta clay
Planting date	Mar. 28, 1963	Aug. 1, 1963	Apr. 18, 1963	July 10, 1963
Harvesting date	July 1, 1963	Nov. 12, 1963	Aug. 15, 1963	Oct. 17, 1963
Age at harvest	13.6 weeks	14.7 weeks	18.4 weeks	14.1 weeks

Fertilizer treatments and mean yields

Treatment No.	Units of P[1]	Y[2]	Y'[3]	Y	Y'	Y	Y'	Y	Y'
1	0.00	46.2	55.0	40.3	47.9	19.6	19.7	34.3	32.0
2	.05	57.9	55.6	53.9	49.1	25.3	20.1	28.1	32.2
3	.10	56.6	56.1	53.9	50.2	21.4	20.5	32.5	32.5
4	.15	58.8	56.7	50.1	51.3	22.4	20.8	37.4	32.7
5	.20	60.6	57.1	54.4	52.4	13.0	21.2	29.1	32.9
6	.50	52.9	59.2	59.9	57.4	21.8	23.1	30.5	33.9
7	.75	62.4	60.0	57.2	59.0	26.7	24.3	34.0	34.1
8	1.00	59.7	59.6	55.5	57.8	22.3	24.8	37.7	33.8
9	1.50	58.5	56.1	50.6	48.8	25.6	23.7	31.3	31.6

Statistics of the fitted equations

A			60.0		59.0		24.8		34.1
B			.14		.39		.23		.13
C			.80		.77		1.06		.72
Sy_ey_e[4]			151.00		118.00		115.00		85.00

[1] 1 unit of P = 1,000 lb. P per acre.

[2] Y = observed mean yield in hundredweights of grain per acre.

[3] Y' = mean yield in hundredweights of grain per acre estimated with the fitted equation.

[4] Sy_ey_e = sum of squares of deviations of the estimated from the observed mean yields.

CALCIUM EXPERIMENTS

Tables 7 and 8 show the data from two sugarcane and four corn experiments carried out with calcium sulfate, in which the calcium applications varied from zero to 4,000 pounds of calcium per acre. Figure 7, A and B gives an idea of the fit obtained with the fertilizer-yield equation for the sugarcane yield data, and figure 8, A, B, C, and D provides similar information as to the corn yield data.

The estimated calcium *B* values for sugarcane are −0.006 and 0.001. For corn, the estimated calcium *B* values are −0.04, 0.004, 0.01, and 0.02. Although, from a strictly statistical point of view, the closeness of the fits obtained in the cases of the aberrant data of the sugarcane experiment at

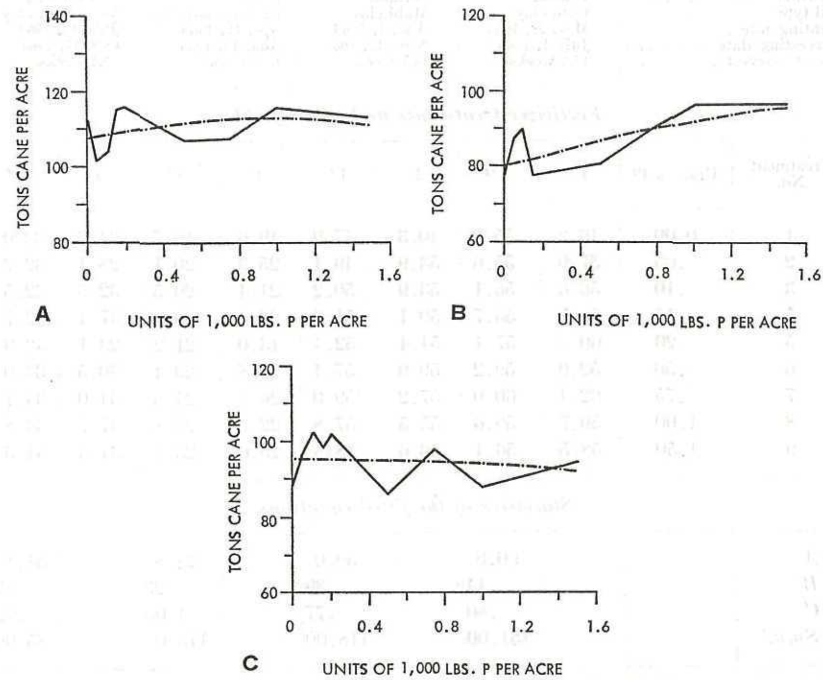

FIG. 3.—Phosphorus-sugarcane relation at: A, Isabela; B, Gurabo; C, Patillas.

Isabela (fig. 7, A), and the corn experiment at Lajas (fig. 8, C) must be considered to be good, the author hopes the readers will agree with him that these negative and very low estimated values of *B* must be interpreted as indicating that the corresponding real values of *B* must be close to zero, but positive. This seems to be more likely on considering the very low positive values of *B* estimated from the data of the other four experiments with calcium.

The estimated optimum calcium applications are 2,000 and 2,700 pounds of calcium per acre for sugarcane and 1,900, 3,000, 3,200, and 3,600 pounds of calcium per acre for corn yields.

The sums of squares of deviations of the estimated from the observed

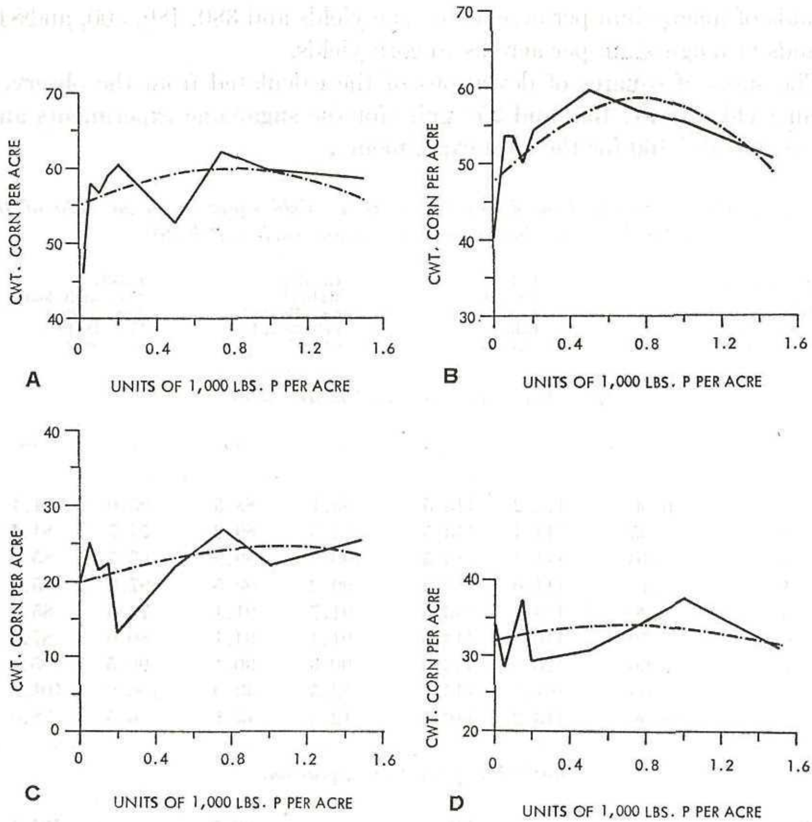

FIG. 4.—Phosphorus-corn relation at: A, Isabela; B, Gurabo; C, Lajas; D, Río Piedras.

mean yields are 248 and 768 units for the sugarcane and 40, 85, 87, and 107 units for the corn data.

MAGNESIUM EXPERIMENTS

Tables 9 and 10 present the data from three sugarcane and four corn experiments with magnesium sulfate, in which the magnesium applications varied from zero to 1,200 pounds of magnesium per acre. Figure 9, A, B, and C indicates the closeness of the fit obtained with the fertilizer-yield equation on the sugarcane yield data, while figure 10, A, B, C, and D provides similar information on the corn yield data.

The estimated magnesium B values are, for sugarcane, 0.09, 0.11, and 0.26; and, for corn, 0.09, 0.29, 0.69, and 0.90.

The estimated optimum magnesium applications are 240, 650, and 970 pounds of magnesium per acre as to cane yields and 380, 480, 500, and 810 pounds of magnesium per acre as to corn yields.

The sums of squares of deviations of the calculated from the observed mean yields are 97, 158, and 259 units for the sugarcane experiments and 36, 41, 69, and 160 for the corn experiments.

TABLE 5.—*Results obtained on fitting the fertilizer-yield equation to the data of the potassium experiments with sugarcane variety P.R. 980*

Location		Isabela		Gurabo		Patillas	
Soil type		Coto clay		Mabí clay		Viví sandy loam	
Planting date		Nov. 28, 1962		Dec. 19, 1962		Nov. 3, 1962	
Harvesting date		Feb. 4, 1964		Feb. 25–7, 1964		Mar. 25, 1964	
Age at harvest		57.4 weeks		62 weeks		69.4 weeks	
Fertilizer treatments and mean yields							
Treatment No.	Units of K[1]	Y[2]	Y'[3]	Y	Y'	Y	Y'
1	0.00	112.2	113.5	89.1	88.5	95.0	84.4
2	.05	114.4	113.5	71.3	89.2	74.5	84.7
3	.10	111.4	113.5	100.9	89.9	93.5	85.0
4	.15	114.0	113.5	99.2	90.5	97.0	85.3
5	.20	116.1	113.5	91.7	91.1	73.5	85.6
6	.50	116.4	113.5	91.4	94.1	80.0	87.4
7	2.00	120.6	113.6	90.3	90.1	90.5	95.5
8	5.00	101.3	113.1	42.5	43.0	106.0	101.3
9	10.00	113.2	110.7	12.3	13.1	76.5	78.0
Statistics of the fitted equations							
A			113.6		96.5		101.5
B			.0004		.08		.01
C			1.9		1.06		4.51
Sy_ey_e[4]			217.00		526.00		676.00

[1] 1 unit of K = 1,000 lb. K per acre.
[2] Y = Observed mean yield in tons of cane per acre.
[3] Y' = Mean yield in tons of cane per acre estimated with the fitted equation.
[4] Sy_ey_e = Sum of squares of deviations of the estimated from the observed mean yields.

DISCUSSION OF RESULTS

The usefulness of any fertilizer-yield equation depends: 1, On its accuracy; 2, on the labor and financial cost of determining the constants needed to permit its practical use under a given crop-production-environmental set-up; and 3, on the cost and labor required to obtain the values that must be fed into the equation to estimate optimum economic applications of fertilizers.

In the particular case of the fertilizer yield equation with which we are now dealing, its usefulness depends, therefore, on 1, its accuracy, 2, the cost and labor involved in obtaining the value of B applicable to a certain fertilizer material for the production of the crop to be produced, and 3,

TABLE 6.—*Results obtained on fitting the fertilizer-yield equation to the data of the potassium experiments with corn variety Mayorbela*

Location	Isabela	Gurabo	Lajas	Río Piedras
Soil type	Coto clay	Mabí clay	Fraternidad clay	Vega Alta clay
Planting date	Mar. 28, 1963	Aug. 1, 1963	Apr. 18, 1963	July 10, 1963
Harvesting date	July 1, 1963	Nov. 12, 1963	Aug. 15, 1963	Oct. 17, 1963
Age at harvest	13.6 weeks	14.7 weeks	18.4 weeks	14.1 weeks

Fertilizer treatments and mean yields

Treatment No.	Units of K[1]	Y[2]	Y'[3]	Y	Y'	Y	Y'	Y	Y'
1	0.00	58.2	54.1	59.9	58.0	11.5	18.3	31.3	32.0
2	.05	56.1	54.2	55.0	58.0	25.7	18.4	29.5	32.3
3	.10	56.6	54.3	59.3	58.0	19.1	18.5	31.5	32.5
4	.15	48.3	54.4	58.3	57.9	25.3	18.6	31.9	32.8
5	.20	48.6	54.5	57.7	57.9	15.3	18.6	37.2	33.0
6	.50	58.3	55.2	60.4	57.7	21.1	19.0	36.3	34.3
7	2.00	59.7	57.8	52.3	54.5	12.5	19.3	33.5	35.6
8	5.00	57.9	59.7	40.3	42.3	20.6	14.2	21.3	20.7
9	10.00	52.9	52.2	25.0	23.8	1.2	6.1	7.5	6.8

Statistics of the fitted equations

A			59.8		58.0		19.5		36.2
B			.005		.014		.03		.06
C			4.6		−.14		1.46		1.47
Sy_ey_e[4]			115.00		32.00		271.00		37.00

[1] 1 unit of K = 1,000 lb. K per acre.

[2] Y = observed mean yield in hundredweights of grain per acre.

[3] Y' = mean yield in hundredweights of grain per acre estimated with the fitted equation.

[4] Sy_ey_e = sum of squares of deviations of the estimated from the observed mean yields.

the labor and cost required to determine the value of C, which must be used in the equation to be able to estimate the optimum application of the fertilizer material. With known values of B and C, and with a suitable estimate of A based on the local experience with the crop in the particular field where it is to be produced commercially, an estimate of the optimum economic fertilizer application may be made.

To estimate B, experiments of the type described here need to be made with the crop and in fields similar to those in which the actual production

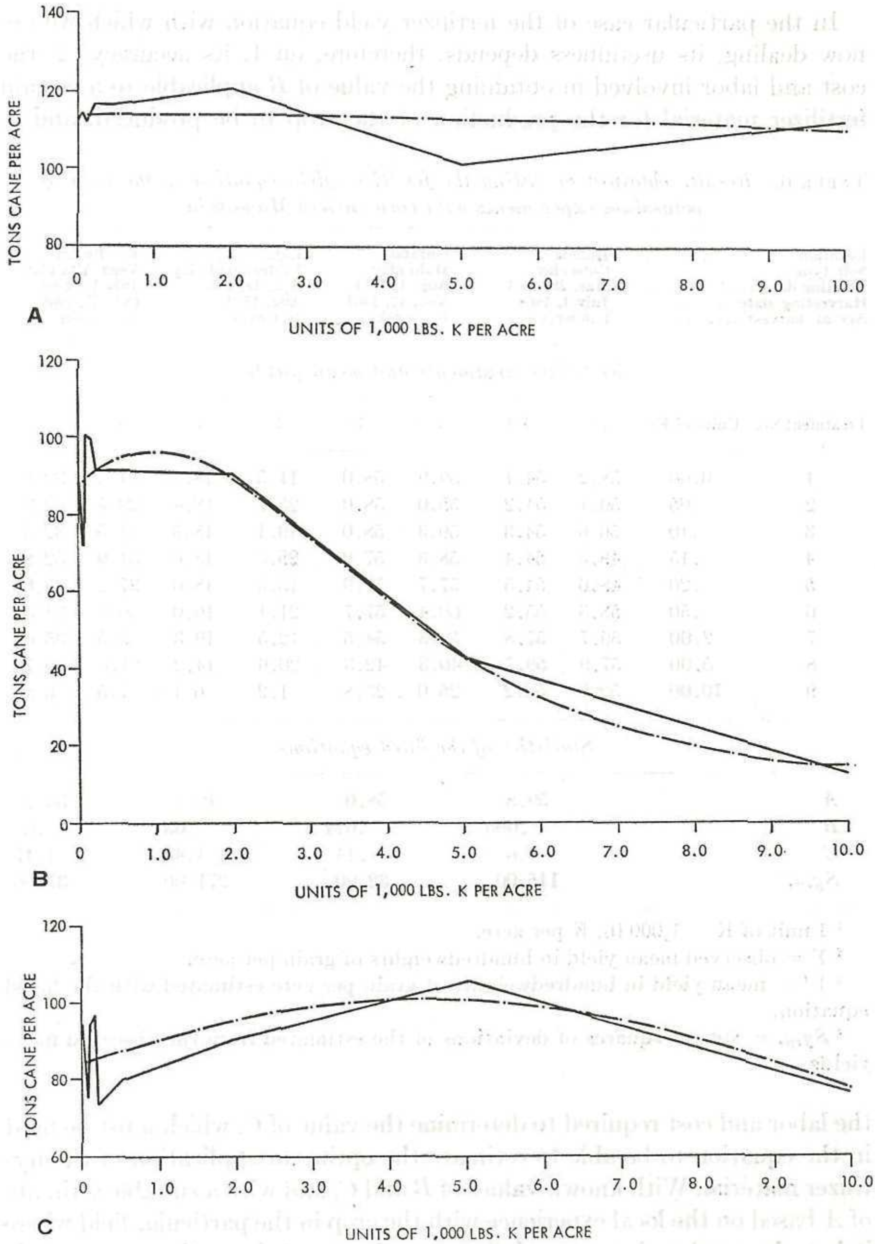

FIG. 5.—Potassium-sugarcane relation at: A, Isabela; B, Gurabo; C, Patillas.

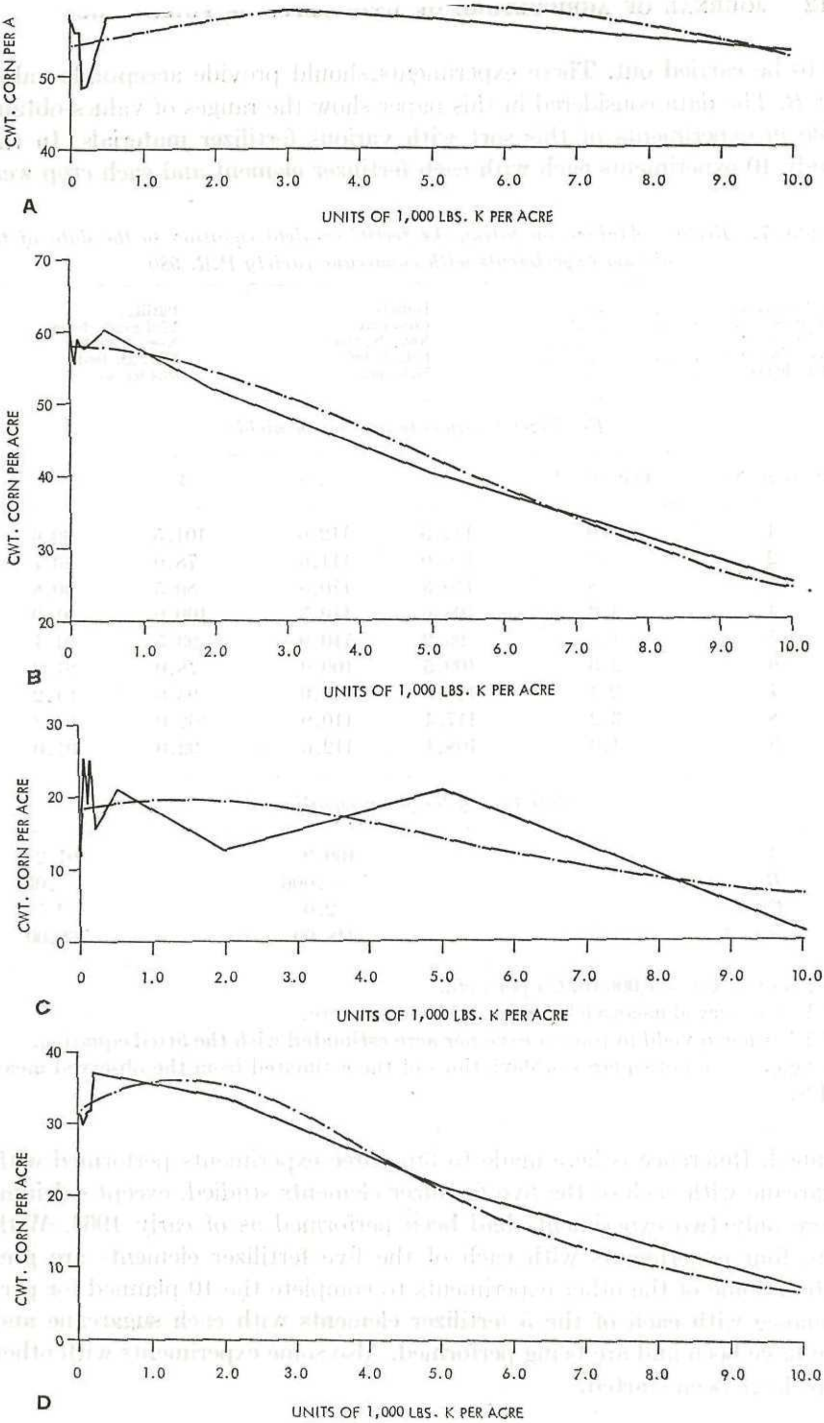

FIG. 6.—Potassium-corn relation at: A, Isabela; B, Gurabo; C, Lajas; D, Río Piedras.

is to be carried out. These experiments should provide acceptable value- for B. The data considered in this paper show the ranges of values obtains able in experiments of this sort with various fertilizer materials. In this study 10 experiments each with each fertilizer element and each crop were

TABLE 7.—*Results obtained on fitting the fertilizer-yield equation to the data of the calcium experiments with sugarcane variety P.R. 980*

Location	Isabela	Patillas
Soil type	Coto clay	Viví sandy loam
Planting date	Nov. 28, 1962	Nov. 3, 1962
Harvesting date	Feb. 4, 1964	Mar. 25, 1964
Age at harvest	57.4 weeks	69.4 weeks

Fertilizer treatments and mean yields

Treatment No.	Units of Ca[1]	Y[2]	Y'[3]	Y	Y'
1	0.0	112.5	112.6	101.5	90.6
2	.4	116.0	111.6	78.0	90.7
3	.8	110.3	110.9	80.5	90.8
4	1.2	108.6	110.3	100.0	91.0
5	1.6	99.2	110.0	93.5	91.1
6	2.0	109.5	109.9	78.0	91.2
7	2.4	116.8	110.0	93.0	91.2
8	3.2	117.4	110.9	102.0	91.2
9	4.0	108.1	112.6	92.0	91.0

Statistics of the fitted equations

A			109.9		91.2
B			−.006		.001
C			2.0		2.7
Sy_ey_e[4]			248.00		768.00

[1] 1 unit of Ca = 1,000 lb. Ca per acre.

[2] Y = observed mean yield in tons of cane per acre.

[3] Y' = mean yield in tons of cane per acre estimated with the fitted equation.

[4] Sy_ey_e = sum of squares of deviations of the estimated from the observed mean yields.

planned. Reference is here made to but three experiments performed with sugarcane with each of the five fertilizer elements studied, except calcium, where only two experiments had been performed as of early 1964. With corn, four experiments with each of the five fertilizer elements are presented. Some of the other experiments to complete the 10 planned for performance with each of the 5 fertilizer elements with each sugarcane and corn have been and are being performed. Also some experiments with other crops have been started.

On reviewing the data presented in this paper, it will be noticed that:

1. The nitrogen and magnesium B values for sugarcane estimated from the experimental data range from 0.09 to 0.32, whereas the phosphorus, potassium, and calcium B values are very low. Should these ranges be confirmed in the other experiments underway, this would mean that sugarcane growers need to exercise a greater care in maintaining the available nitrogen and magnesium contents of their fields as close as possible to their respective optimum concentrations than for phosphorus, potassium, and calcium. This would be so because sugarcane production would tend to suffer more from a lack of equilibrium in the available nitrogen and magnesium contents of the soil than from that of phosphorus, potassium, and calcium.

TABLE 8.—*Results obtained on fitting the fertilizer-yield equation to the data of the calcium experiments with corn variety Mayorbela*

	Isabela	Gurabo	Lajas	Río Piedras
Location	Isabela	Gurabo	Lajas	Río Piedras
Soil type	Coto clay	Mabí clay	Fraternidad clay	Vega Alta clay
Planting date	Mar. 28, 1963	Aug. 1, 1963	Apr. 18, 1963	July 10, 1963
Harvesting date	July 1, 1963	Nov. 12, 1963	Aug. 15, 1963	Oct. 17, 1963
Age at harvest	13.6 weeks	14.7 weeks	18.4 weeks	14.1 weeks

Fertilizer treatments and mean yields

Treatment No.	Units of Ca[1]	Y[2]	Y'[3]	Y	Y'	Y	Y'	Y	Y'
1	0.0	55.3	57.0	37.6	41.2	31.6	30.4	37.1	35.2
2	.4	61.9	57.5	47.4	43.1	26.9	28.6	37.3	36.0
3	.8	59.6	58.0	46.3	44.9	31.3	27.3	35.6	36.7
4	1.2	52.7	58.3	47.4	46.5	22.6	26.5	38.6	37.3
5	1.6	56.1	58.6	47.4	48.1	22.0	26.1	32.9	37.8
6	2.0	63.5	58.9	49.0	49.4	24.3	26.0	35.1	38.3
7	2.4	58.3	59.0	48.5	50.4	28.7	26.3	42.7	38.6
8	3.2	58.2	59.1	51.2	51.7	34.1	27.9	43.4	38.8
9	4.0	60.4	58.9	52.8	51.7	28.8	31.6	36.3	38.6

Statistics of the fitted equations

A			59.1		51.9		26.0		38.8
B			.004		.02		−.04		.01
C			3.0		3.6		1.9		3.2
Sy_ey_e[4]			87.00		40.00		107.00		85.00

[1] 1 unit of Ca = 1,000 lb. Ca per acre.

[2] Y = observed mean yield in hundredweights of grain per acre.

[3] Y' = mean yield in hundredweights of grain per acre estimated with the fitted equation.

[4] Sy_ey_e = sum of squares of deviations of the estimated from the observed mean yields.

2. The *nitrogen B* values for corn estimated from the available experimental data range from 0.10 to 0.70; the phosphorus *B* values range from 0.13 to 0.39; and the magnesium *B* values range from 0.09 to 0.90; while the estimated potassium and calcium values again, in general, are very

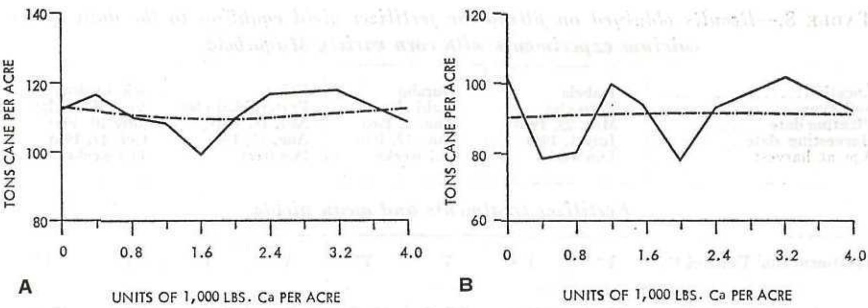

FIG. 7.—Calcium-sugarcane relation at: A, Isabela; B, Patillas.

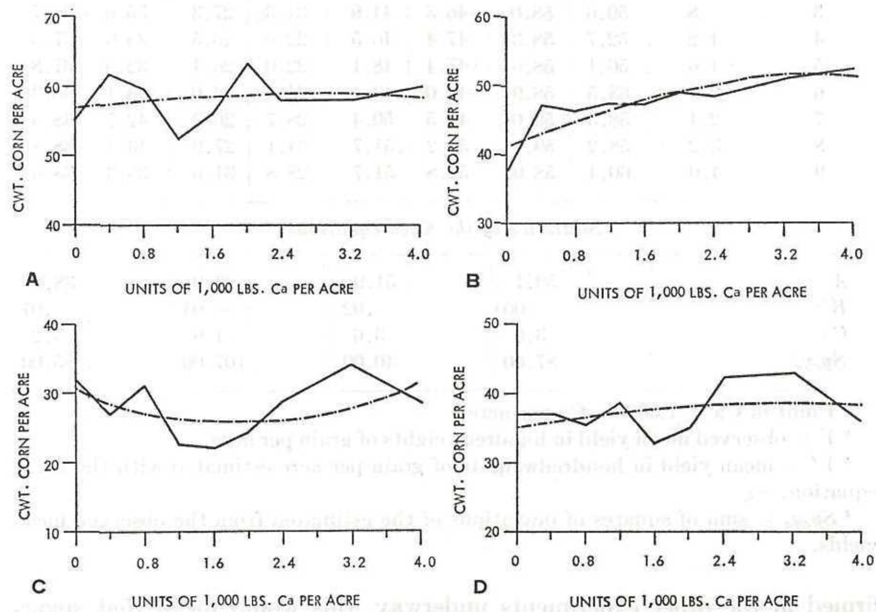

FIG. 8.—Calcium-corn relation at: A, Isabela; B, Gurabo; C, Lajas; D, Río Piedras.

low. For corn production in soils similar to the ones of the experimental fields used for these experiments, therefore, growers need pay more attention to the available nitrogen, phosphorus, and magnesium soil status than to the corresponding potassium and calcium status. This is true because, according to the above-mentioned values, corn production seems to be influenced to a greater degree by a lack of equilibrium in the soil as regards available contents of nitrogen, phosphorus, and magnesium than is the case with potassium and calcium.

The estimation of the value C, that is, by how much does the fertilizer status of the soil differ from the optimum as regards content of the corresponding fertilizer material, has also been a subject on which there has been no consensus among its students. In a previous article by the author (*2*) a scheme was suggested to estimate the optimum fertilizer application to use with a given crop. The suggested scheme was based on a relationship discovered by the author (*4*) between the nutrient content of the leaves of the plants of a crop and the expected relative yield of the crop. With the nutrient contents of the leaves known, therefore, it should be possible to estimate how far from the optimum nutrient level the status of the soil is where the crop is grown. When the above-mentioned paper (*2*)

TABLE 9.—*Results obtained on fitting the fertilizer-yield equation to the data of the magnesium experiments with sugarcane variety P.R. 980*

Location	Isabela	Gurabo	Patillas
Soil type	Coto clay	Mabí clay	Viví sandy loam
Planting date	Nov. 28, 1962	Dec. 19, 1962	Nov. 3, 1962
Harvesting date	Feb. 4, 1964	Feb. 25–7, 1964	Mar. 25, 1964
Age at harvest	57.4 weeks	62 weeks	69.4 weeks

Fertilizer treatments and mean yields

Treatment No.	Units of Mg[1]	Y[2]	Y'[3]	Y	Y'	Y	Y'
1	0.00	109.2	104.3	94.3	89.7	75.5	73.2
2	.12	101.5	105.7	82.1	90.0	82.0	76.7
3	.24	106.5	106.7	95.9	90.2	78.5	80.0
4	.36	104.4	107.4	86.1	90.0	69.5	83.0
5	.48	105.8	108.0	87.6	89.7	88.5	85.8
6	.60	114.5	108.3	88.5	89.0	89.0	87.9
7	.72	107.9	108.3	92.1	88.0	94.5	89.7
8	.96	105.5	107.3	87.1	85.3	92.5	91.1
9	1.20	106.0	105.5	80.4	81.9	87.0	89.8

Statistics of the fitted equations

A			108.3		90.2		91.1
B			.09		.11		.26
C			.65		.24		.97
Sy_ey_e[4]			97.00		158.00		259.00

[1] 1 unit of Mg = 1,000 lb. Mg per acre.

[2] Y = observed mean yield in tons of cane per acre.

[3] Y' = mean yield in tons of cane per acre estimated with the fitted equation.

[4] Sy_ey_e = Sum of squares of deviations of the estimated from the observed mean yields.

was prepared, the weakest link in the chain of the scheme seemed to be the accuracy of the fertilizer-yield equation. It seems that this link has become considerably strengthened by the data presented herein.

The weakest link has become now the accuracy of the relationship between the nutrient content of the leaves and the relative yield of the crop. This is so because the arc-tangent relationship previously suggested by the author (*4*) was based on data obtained from experiments in which the maximum rates of application of the fertilizer elements studied were well below the rates considered to be harmful to crop production.

TABLE 10.—*Results obtained on fitting the fertilizer-yield equation to the data of the magnesium experiments with corn variety Mayorbela*

Location	Isabela	Gurabo	Lajas	Río Piedras
Soil type	Coto clay	Mabí clay	Fraternidad clay	Vega Alta clay
Planting date	Mar. 28, 1963	Aug. 1, 1963	Apr. 18, 1963	July 10, 1963
Harvesting date	July 1, 1963	Nov. 12, 1963	Aug. 15, 1963	Oct. 17, 1963
Age at harvest	13.6 weeks	14.7 weeks	18.4 weeks	14.1 weeks

Fertilizer treatments and mean yields

Treatment No.	Units of Mg[1]	Y[2]	Y'[3]	Y	Y'	Y	Y'	Y	Y'
1	0.00	50.3	51.3	56.1	56.5	28.8	26.4	36.5	35.8
2	.12	58.8	54.7	56.2	57.0	23.7	27.6	38.8	38.1
3	.24	55.6	57.0	61.5	57.4	33.3	28.7	36.2	39.9
4	.36	54.8	58.0	57.7	57.6	31.0	29.7	43.4	41.1
5	.48	56.6	57.5	53.9	57.7	21.6	30.4	41.2	41.5
6	.60	58.8	55.6	57.2	57.6	32.1	31.0	43.8	41.1
7	.72	52.4	52.5	58.8	57.5	33.8	31.3	39.1	39.9
8	.96	44.3	44.5	55.5	56.6	35.4	31.2	30.6	35.8
9	1.20	36.0	36.1	56.1	55.3	26.4	30.1	34.4	30.6

Statistics of the fitted equations

A			58.0		57.7		31.4		41.5
B			.90		.09		.29		.69
C			.38		.50		.81		.48
Sy_ey_e[4]			41.00		36.00		160.00		69.00

[1] 1 unit of Mg = 1,000 lb. Mg per acre.

[2] *Y* = observed mean yield in hundredweights of grain per acre.

[3] *Y'* = mean yield in hundredweights of grain per acre estimated with the fitted equation.

[4] Sy_ey_e = sum of squares of deviations of the estimated from the observed mean yields.

In this connection it should be mentioned that, in the study underway at the Agricultural Experiment Station of the University of Puerto Rico, leaf samples have been taken of the experimental crops to study more

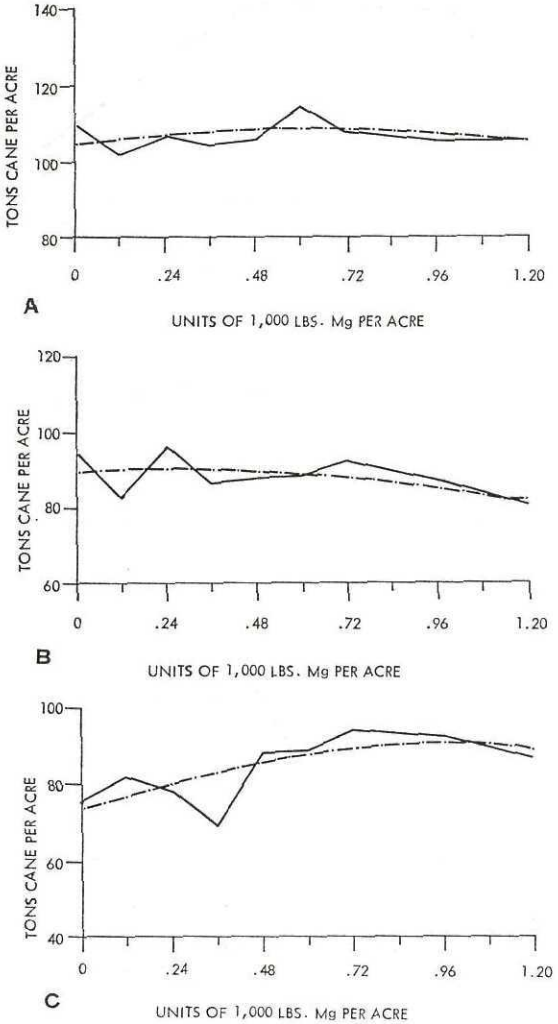

FIG. 9.—Magnesium-sugarcane relation at: A, Isabela; B, Gurabo; C, Patillas.

thoroughly the usefulness of the data relative to the nutrient contents of leaves. Preliminary studies of these data suggest that a Mitscherlich's-type equation may be used to express the relation between available nutrient contents of the leaves and the soil.

If this suggestion is confirmed by future work on this subject, it may be possible to estimate the soil's available nutrient status from the chemical analysis of a leaf sample, and then, using this estimated value of the soil's available nutrient content in the fertilizer-yield equation, to estimate the

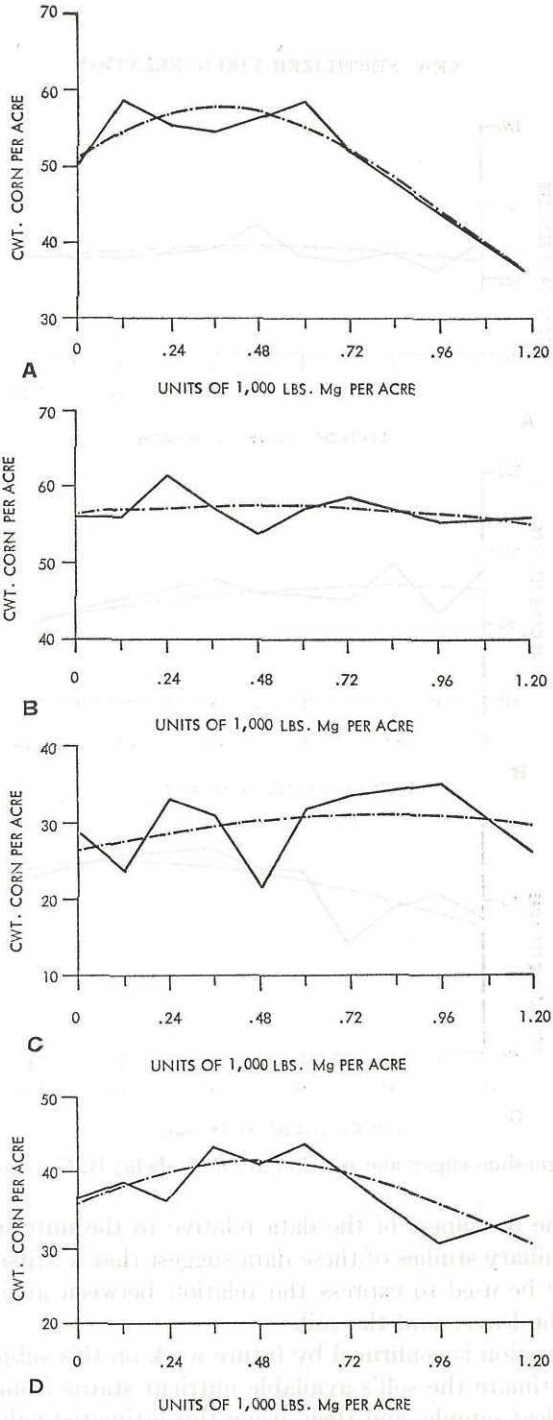

FIG. 10.—Magnesium-corn relation at: A, Isabela; B, Gurabo; C, Lajas; D, Río Piedras.

crop-yield response to applications of the corresponding fertilizer material, and the optimum economic fertilizer application.

SUMMARY

A new fertilizer-yield equation recently discovered by the author was used to describe the relation between the applications of nitrogen-, phosphorus-, potassium-, calcium-, and magnesium-bearing materials and the yields of sugarcane and corn in 34 experiments carried out in various regions of Puerto Rico near Isabela, Gurabo, Patillas, Lajas, and Río Piedras. The equation under study satisfactorily fitted such fertilizer-yield data, even in cases where the fertilizer-yield equation suggested by other workers do not apply. The general applicability of the new fertilizer-yield equation even in cases of certain abnormal data has been noted.

From the data studied, it seems that, as concerns the production of sugarcane in soils similar to those used in the above-mentioned experiments, there is more need to keep the available soil nitrogen and magnesium contents as near as possible to their optimum equilibrium values than is true of phosphorus, potassium, and calcium. In corn production the equilibrium of the available nitrogen, phosphorus, and magnesium seems to be more important than that of the available potassium and calcium.

RESUMEN

Se utilizó una nueva ecuación abono-rendimiento descubierta recientemente por el autor para describir la relación entre la aplicación de materiales que contienen nitrógeno, fósforo, potasio, magnesio y calcio y el rendimiento de caña y maíz en 34 experimentos de campo llevadosa cabo en varias regiones de Puerto Rico cerca de Isabela, Gurabo, Patillas, Lajas y Río Piedras. Los resultados demostraron que la ecuación abono-rendimiento describió satisfactoriamente la relación antes indicada, aun en aquellos casos en que las relaciones abonorendimiento sugeridas por otros investigadores no aplican. La utilidad general de la ecuación sugerida, aun en casos de datos "anormales", ha sido señalada.

De los datos estudiados tal parece que, en cuanto a la producción de caña se refiere, en terrenos similares a los utilizados en los experimentos mencionados, es más necesario el mantener los niveles asimilables en los casos de nitrógeno y magnesio en el suelo tan cerca como sea posible a sus niveles óptimos correspondientes, que en los del fósforo, del potasio y del calcio. En cuanto a la producción de maíz, los equilibrios de los contenidos asimilables de nitrógeno, fósforo y magnesio en el suelo son más importantes que los del potasio y del calcio.

LITERATURE CITED

1. Balmukand, Bh., The relation between yield and soil nutrients, *J. Agr. Sci. 18:* 602–25, 1928.

2. Capó, B. G., A new approach to the problem of adequate fertilization of a crop, *J. Agr. Univ. P.R. 46*(4): 243–53, 1962.
3. ———, A new fertilizer-yield equation, Communication to the XII Congr. of the Int. Soc. Sugarcane Tech., held in San Juan, P.R., March 1965.
4. Capó, B. G., and Samuels, G., The development of a mathematical concept to interpret the relation between plant composition and crop yield, *J. Agr. Univ. P.R. 37*(4): 249–64, 1955.
5. Lang, E., The Law of the Soil, The Law of Diminishing Returns, Part 2, World Book Co., Chicago, Ill., 1924.
6. Mitscherlich, E. A., Die Bestimmung des Dungerbedurfnisses des Bodens, Paul Parey, Berlin, Germany, 1931.
7. Russel, E. J., Soil Conditions and Plant Growth, 7th ed., Longmans, Green & Co., New York, N.Y., 1937.
8. Sapehin, A. A., Zum Gesetz des Ertrages, *Deutsche Botanische Gesellschaft 41*: 386–90, 1923.
9. Spillman, W. J., Use of the exponential yield curve in fertilizer experiments, USDA Tech. Bull. 348, 1933.