Research Note

A COMPUTER PROGRAM FOR THE ANALYSIS OF EXPERIMENTAL DATA[1]

In a recently published book on applied statistics,[2] the junior author described a generalized procedure for the analysis of research data. The procedure consists of: 1) formulating the linear regression equation which corresponds to the design used in collecting the data; 2) preparing, in tabular form, the observation equations; 3) calculating the corrected sums of squares and cross products, which constitute the matrix of coefficients of the statistics in the normal equations; 4) solving the normal equations by a matrix-inversion method developed by the author of the book; 5) calculating the values of the statistics representing the influences of the factors under study; 6) evaluating the portion of the sum of squares of the dependent factor explained by the independent factors; and 7) testing the significance of the corresponding statistics or of their differences.

The procedure applies to the study of the influence of both qualitative and quantitative factors from data collected with either balanced or unbalanced designs. A qualitative factor is one whose levels constitute a finite group of mutually exclusive possibilities, such as the blocks or treatments in a complete block design. A quantitative factor is one with an infinite number of levels, such as the weight of an animal. When such a factor is an external source of variation, included in the analysis to eliminate its influence, it is referred to as a covariance factor.

In this connection, a computer program has been developed, which, starting with the observed data, prepares the corresponding observation and normal equations, inverts the matrix of the coefficients of the normal equations, and provides the corresponding tables of analysis of variance and tests of significance of the differences between the treatment means.

The program can handle data collected for 3 qualitative factors, i.e. treatments, blocks, and replications; and up to 8 quantitative factors. As used herein, the program can evaluate up to 100 independent statistics representative of the influences of the quantitative factors or of the levels of the qualitative factors. It can be easily expanded without a marked increase of the memory requirements for its storage. The program is currently used in an 8 K words IBM-1130 computer system.

For matrix inversion of the coefficients of the normal equations, the program uses an algorithm, also developed by the junior author,[2] which requires less memory area in the computer than most matrix inversion

[1] Manuscript submitted to Editorial Board January 31, 1978.

[2] Capó, B. G., 1977. Análisis estadístico de datos de investigaciones, B. G. Capó, San Juan, P.R.

methods in use. This is due to the fact that, as matrix inversion goes on, the original and intermediate matrices are destroyed and their memory locations used for the storage of the elements of the inverse.

The program is designed for printing the following information: regression coefficients; treatment means and predicted values; the analysis of variance table, showing the reduction in the sum of squares corresponding to each factor as it is introduced; and the t test of significance for the statistical comparisons of the ranked treatment means, at both the 5- and 1-% levels of probability. It may also optionally print the following information: observation equations, matrix of corrected sums of squares and cross products, and the Gauss multipliers or elements of the inverted matrix.

The initial setup of the card deck should be in the following order:

1. Main program, subroutines, and subprogram cards with the corresponding system control cards.
2. Problem description cards.
   There are two problem description cards in front of the data cards. The first card is for problem identification only, and may be left in blank. The second card specifies the number of levels for each of the qualitative factors to be included in the regression, the number of data cards (observations), and other information such as that relative to the mathematical model used and the output options.
3. Data cards
   There is one card per observation. The first 10 spaces, used for job identification, are skipped by the program. Three integer data fields follow for the replication, block, and treatment numbers corresponding to the particular observation. After 4 more spaces, data fields are defined for the location of the observed values of the different dependent variables and covariance factors.
4. Cards with t test theoretical values.
   A set of cards with the appropriate t values according to the degrees of freedom for error of the problem follows the data cards. The program will read first the 5% probability level values, 12 values/card, up to NT-1 values, where NT is the number of treatments; or 39, if there are more than 40 treatments. The 1% probability level values are next read in the same way. (The t values used were calculated multiplying the corresponding Duncan's multiple range values by $\sqrt{}2$).[3]

[3] Harter, Leon H., 1960. Critical values for Duncan's new multiple range test, Biometrics, 16:(4) 671–85.

The program listing is as follows:

```
PAGE    2                                   NAME.... MAIN2

      DEFINE FILE 1(5050,3,U,IJ)
      DIMENSION S(100),X(100),SX(100),RMEAN(100),IDENT(20),NOT(100)
      DIMENSION FLINE(20),LCOV(8)
      COMMON  PROM   ,NVI,LC1,NC,NT,GIE,X,VE
      MAXIMUM CAPACITY .... 100 REGRESSION COEFFICIENTS
      FLINE - 1ST. CARD , FOR JOB DESCRIPTION
      IDENT - JOB DESCRIPTION

      NT    =  NUMBER OF TREATMENTS
      NB    =  NUMBER OF BLOCKS
      NR    =  NUMBER OF REPLICATIONS
      NO    =  NUMBER OF OBSERVATIONS
      IST   =  NUMBER OF INDEPENDENT FACTORS
                    IF 1 IS SPECIFIED THE SUMS OF SQUARES FOR BOTH
                    QUALITATIVE AND QUANTITATIVE VARIABLES ARE POOLED
      LY    =  LOCATION OF THE DEPENDENT VARIABLE
      MC       = 0 TO CALCULATE THE GAUSS MULTIPLIERS
               = 1 IF MULTIPLIERS ARE ALREADY STORED
      NCOV =  NUMBER OF QUANTITATIVE FACTORS
      LCOV =  LOCATION OF THE QUANTITATIVE FACTORS.
      LSQ  =  1  FOR LATIN SQUARES DESIGN , NR WILL CORRESPOND TO ROWS
                    AND NB TO COLUMNS

     SW1 'ON'     - TO PRINT OBSERVATION EQUATIONS
     SW3 'ON'     - TO PRINT TABLES OF CROSS PRODUCTS AND GAUSS MULTIPLIERS
     SW4 'OFF'    - TO PRINT SIGNIFICANT DIFFERENCES ONLY

      READ (2,25) FLINE
   25 FORMAT (20A4)
      READ (2,3) NT,NB,NR,NO,IST,LY,MC,NCOV,LCOV,LSQ,IDENT
    3 FORMAT (3I2,I3,I1,I2,11I1,17X,20A2)
      WRITE (1,100) NT,NB,NR,NO,LY,NCOV,MC,(LCOV(I),I=1,NCOV)
  100 FORMAT (3X,'NT    NB    NR    NO    LY    NCOV MC  LC1  LC2  LC3 ...'
     1,/(9I5)/)
      LC=LY
      CALL DATA2 (NT,NB,NR,NO,LY,NCOV,LC1,LC2,LCOV,MC,S,X,SX,NOT,RMEAN,
     1LSQ)
      NVI=LY
      IO=1
      CALL DATSW (3,IO)
      GO TO (5,7),IO
    5 WRITE (3,6)
    6 FORMAT (/35X,'MATRIX OF CORRECTED SQUARES AND CROSS PRODUCTS'/35X,
     1'SAA,SAB,SBB,SAC,SBC,SCC,....SNN'/)
      LC=LC1
      GO TO 48
    7 CONTINUE
      K=LC1-LY
      PROM=SX(NVI)/NO
      CALL SOL2(S,X,SX,LY,LC1,LC2,MC,NO,NT,NB,NR,IST,VE,R,LSQ)
      NCO=NVI-1
      IO=2
      GO TO (8,11),IO
    8 WRITE (3,9)
    9 FORMAT(/35X,'MATRIX OF GAUSS MULTIPLIERS'/35X,'CAA,CAB,CBB,CAC,CBC
     1,CCC, ...CNN'/)
      LC=K
```

```
PAGE    3                                   NAME.... MAIN2

   48 IR=10
      IM=LC /IR+1
      IJ=1
      DO 52 I=1,IM
      IF (I-IM) 54,49,54
   49 IR=LC -IR*(IM-1)
   54 READ (1'IJ) (S(IN),IN=1,IR)
   52 WRITE (3,53) (S(IN),IN=1,IR)
   53 FORMAT (1X,10F11.6)
      GO TO (7,11),ID
   11 WRITE (3,30) FLINE,IDENT
   30 FORMAT (/2X,20A4/35X,20A2/)
      WRITE (3,12) (X(I),I=1,NCO)
   12 FORMAT (35X,'MATRIX OF REGRESSION COEFFICIENTS'/35X,
     1'T(I) , B(J) , R(K) , COV(L)'//(2X,10F10.4))
      X(NT)=0
      WRITE (3,16)
   16 FORMAT (//1X,'TREATMENT'2X,'REP',9X,'MEAN',6X,'T-COEF',6X,
     1'PREDICTION'//)
      DO 17 I=1,NT
      RMEAN(I)=RMEAN(I)/FLOAT(NOT(I))
      PRO=PROM+X(I)
      WRITE (3,15) I,NOT(I),RMEAN(I),X(I),PRO
   15 FORMAT (2X,I5,3X,I5,1X,3F13.4)
   17 X(NT)=X(NT)-X(I)
      WRITE (3,18) PROM
   18 FORMAT (/4X,'GENERAL MEAN ='F11.4//)
      WRITE (3,19)
   19 FORMAT (/35X,'ANALYSIS OF VARIANCE        '//)
      WRITE (3,20) (SX(J),J=1,IST)
   20 FORMAT (1X,'SOURCE     DF',9X,'SUM OF SQUARES 1/',9X,
     1'MEAN SQUARE'7X,'F 2/',//(10X,F4.0,5X,F15.4,12X,F12.4,4X,F8.2))
      WRITE (3,200)
  200 FORMAT (/1X,'1/ REDUCTION IN SUM OF SQUARES AS EACH FACTOR IS INTR
     1ODUCED'/1X,'2/ F TEST VALID FOR BALANCED DESIGNS,APPROXIMATE FOR U
     2NBALANCED ONES.')
      VE=SX(IST)
      GIE=IST-2
      KL=0
      SSS=SQRT(VE)
      CV=(SSS/PROM)*100.0
      WRITE (3,300) SSS,CV
  300 FORMAT (//1X,'STANDARD DEVIATION =',F15.4/1X,'COEFFICIENT OF VARIA
     1BILITY =',F9.4)
      CALL LINK (TEST2)
      END
```

```
PAGE    2                                NAME .....      DATA2

      SUBROUTINE DATA2 (NT,NB,NR,NO,LY,NCOV,LC1,LC2,LCOV,MC,S,X,SX,
     1NOT,RMEAN,LSQ)
      DIMENSION S(1),SX(1),X(1),VAR(8),NOT(1),RMEAN(1),LCOV(8)
      CALL DATSW (1,IKA)
      GO TO (53,55),IKA
   53 WRITE (3,54)
   54 FORMAT (/35X,'OBSERVATION EQUATIONS'/35X,'Y(I,J,K,L) , T(I) , B(J)
     1 , R(K) , COV(L)'//)
   55 BL=0
      LN=LY+NCOV
      DO 1 N=1,NT
      RMEAN(N)=0.0
    1 NOT(N)=0
      IF (NB) 50,2,3
    2 NB=1
    3 IF(NR) 50,5,6
   50 PAUSE 7777
      CALL EXIT
    5 NR=1
    6 LS=0
      IF (LSQ-1) 21,22,21
   21 LSQ=NR
      LS=1
   22 NTI=NT-1
      NBI=NB-1
      NRI=NR-1
      NVI=NTI+NBI*LSQ+NR+NCOV
      ND=(NVI*(NVI+1))/2.0
      LMC=1
      LM=1
      LT=NVI-1
      IF(MC) 50,8,7
    7 LMC=ND-NVI+1
      LM=NVI
    8 KR=NTI+LSQ*NBI
      DO 9 J=1,NVI
    9 SX(J)=0.0
      DO 11 J=LMC,ND
   11 WRITE (1'J) BL
      DO 100 KJ=1,NO
      L=LMC-1
      DO 12 J=1,NVI
   12 X(J)=0.0
      READ (2,13) IR,IB,IT,VAR
   13 FORMAT(10X,3I2,4X,8F6.2)
      IF (NB-1) 50,31,16
   16 IF (NR-1) 50,14,15
   14 IR=1
      GO TO 23
   15 IF (NR-IR) 50,17,19
   17 DO 18 J=1,NRI
      K=KR+J
   18 X(K)=-1.0
      GO TO 23
   19 K=KR+IR
      X(K)=1.0
   23 KB=NTI+(IB-1)*NBI*LS
```

```
PAGE    3                              NAME .....        DATA2

      IF (NB-IB) 50,25,27
   25 DO 26 J=1,NBI
      K=KB+J
   26 X(K)=-1.0
      GO TO 31
   27 K=KB+IB
      X(K)=1.0
   31 IF(NT-IT) 50,33,35
   33 DO 34 I=1,NTI
   34 X(I)=-1.0
      GO TO 36
   35 X(IT)=1.0
   36 X(NVI)=VAR(LY)
      IF (NCOV) 40,40,37
   37 L1=NVI-NCOV
      DO 38 IDM=1,NCOV
      NN=LCOV(IDM)
      X(L1)=VAR(NN)
   38 L1=L1+1
   40 RMEAN(IT)=RMEAN(IT)+X(NVI)
      NOT(IT)=NOT(IT)+1
      DO 39 I=1,NVI
   39 SX(I)=SX(I)+X(I)
      DO 42 J=LM,NVI
      IF (X(J)) 43,42,43
   43 IS=L+1
      IJ=IS
      READ (1'IJ) (S(IN),IN=1,J)
      FIND (1'IS)
      DO 41 I=1,J
   41 S(I)=S(I)+X(I)*X(J)
      IJ=IS
      WRITE (1'IJ) (S(IN),IN=1,J)
   42 L=L+J
      CALL DATSW (1,IKA)
      GO TO (45,47),IKA
   45 WRITE (3,46) X(NVI),(X(I),I=1,LT)
   46 FORMAT (1X,F9.2,35F3.0/(10X,35F3.0))
   47 CONTINUE
  100 CONTINUE
      L=LMC
      CN=NO
      DO 51 J=LM,NVI
      SXJ=SX(J)/CN
      READ (1'L) (S(IN),IN=1,J)
      FIND (1'L)
      DO 49 I=1,J
   49 S(I)=S(I)-SX(I)*SXJ
      WRITE (1'L) (S(IN),IN=1,J)
      L=L+J
   51 FIND(1'L)
      LY=NVI
      LC1=ND
      LC2=LMC
      RETURN
      END
```

```
PAGE    5                                   NAME .....        SOL2

      SUBROUTINE SOL2 (S,A,G,NVI,ND,LMC,MC,NO,NT,NB,NR,IST,VE,R,LSQ)
      DIMENSION S(1),A(1),G(1),SSR(100),H(100),IS(7)
      NCA=ND-NVI+1
      IF (MC) 3,3,4
    3 MC=2
      READ (1'1) SUM
      SUM=1./SUM
      WRITE (1'1) SUM
      S(1)=SUM
      GO TO 6
    4 MC=NVI
    6 READ (1'NCA) (H(J),J=1,NVI)
      DO 300 K=MC,NVI
      LA=K-1
      E=0
      SSR(LA)=0
      M=K*(K-1)/2.0+1
      READ (1'M) (G(J),J=1,K)
      DO 50 J=1,LA
   50 A(J)=0
      ID=1
      DO 200 J=1,LA
      READ (1'ID) (S(IN),IN=1,J)
      ID=ID+J
      FIND (1'ID)
      DO 100 I=1,J
      IF (I-J) 90,180,90
   90 SSR(LA)=SSR(LA)+2.0*S(I)*H(J)*H(I)
      A(I)=A(I)+S(I)*G(J)
      A(J)=A(J)+S(I)*G(I)
  100 CONTINUE
  180 A(J)=A(J)+S(J)*G(J)
  200 SSR(LA)=SSR(LA)+S(J)*H(J)*H(J)
      DO 220 J=1,LA
  220 E=E+A(J)*G(J)
      R=G(K)-E
      IF (K-NVI) 213,225,225
  213 SN=1.0/R
      G(K)=SN
      DO 240 I=1,LA
  240 G(I)=-SN*A(I)
      WRITE (1'M) (G(IN),IN=1,K)
      ID=1
      DO 300 J=1,LA
      READ (1'ID) (S(IN),IN=1,J)
      FIND (1'ID)
      DO 280 I=1,J
  280 S(I)=S(I)+G(I)*G(J)*R
      WRITE (1'ID) (S(IN),IN=1,J)
  300 ID=ID+J
  225 VE=R/FLOAT(NO-NVI)
      NODE=0
      IS(1)=NT-1
      IS(2)=IS(1)+(NB-1)*LSQ
      IS(3)=IS(2)+NR-1
      IF (NR-1) 22,22,23
   22 IS(3)=IS(3)+1
```

```
PAGE    6                                    NAME .....         SOL2

   23 DO 24 IK=4,7
   24 IS(IK)=IS(IK-1)+1
      IF (NC-2) 27,27,26
   26 IST=1
      IS(1)=NVI-1
   27 CONTINUE
      DO 315 LS=1,IST
      NODE=NODE+1
      N2=IS(LS)
      N3=LS-1
      IF(N3) 31,31,33
   31 SCR=SSR(N2)
      N1=0
      GO TO 35
   33 N1=IS(N3)
      SCR=SSR(N2)-SSR(N1)
   35 AN1=N1
      AN2=N2
      G(NODE)=AN2-AN1
      NODE=NODE+1
      G(NODE)=SCR
      NODE=NODE+1
      G(NODE)=SCR/(AN2-AN1)
      NODE=NODE+1
  315 G(NODE)=SCR/((AN2-AN1)*VE)
      NODE=NODE+1
      GIE=NO-NVI
      G(NODE)=GIE
      NODE=NODE+1
      G(NODE)=R
      NODE=NODE+1
      G(NODE)=VE
      IST=NODE
      RETURN
      END
```

```
PAGE   2                              NAME      TEST2

      DEFINE FILE 1(5050,3,U,IJ)
      DIMENSION S(100),A(100),P(100),XP(100),IT(100),X(100)
      DIMENSION TTAB5(40),TTAB1(40),ID(100),IDENT(3)
      COMMON PG,  NC,NT,KL,NTA,GIE,A,VE
      DATA IDENT/'  ',' *','**'/
C     VECTOR S NO OF USA
      II=0
      NTI=NTA-1
      L1=1
      L2=NTA-1
      K=I+L2
      ISA=(L2*NTA/2)+1
      A(NTA)=0
      DO 4 I=1,NTA
    4 S(I)=0
      IA=1
      DO 180 J=1,NTI
      READ (1'IA) (X(IN),IN=1,J)
      IA=IA+J
      DO 100 I=1,J
      IF (I-J) 90,180,90
   90 S(I)=S(I)-X(I)
  100 S(J)=S(J)-X(I)
  180 S(J)=S(J)-X(J)
      DO 25 I=1,NTI
      A(NTA)=A(NTA)-A(I)
   25 S(NTA)=S(NTA)-S(I)
      READ (1'ISA) (X(JO),JO=1,NTA)
      WRITE (1'ISA) (S(JO),JO=1,NTA)
      DO 66 NI=1,NTA
      IT(NI)=NI
   66 P(NI)=PG+A(NI)
      DO 11 I=1,NTA
      DO 14 J=I,NTA
      IF(P(I)-P(J))12,14,14
   12 PMAX=P(J)
      ITMAX=IT(J)
      P(J)=P(I)
      IT(J)=IT(I)
      P(I)=PMAX
      IT(I)=ITMAX
   14 CONTINUE
   11 XP(I)=P(I)
      WRITE (3,101)
  101 FORMAT (//25X,'TEST OF SIGNIFICANCE'//1X,'TREATMENT',3X,
     1'PREDICTED MEAN'/)
      DO 54 I=1,NTA
      KI=IT(I)
      ID(KI)=I
   54 WRITE (3,120) IT(I),XP(I)
  120 FORMAT (1X,I6,3X,F14.4)
      WRITE (3,113)
  113 FORMAT (//1X,'COMPARISONS',5X,'STANDARD ERROR',4X,1HT/1X,
     1'BETWEEN TREAT.',2X,'OF A DIFFERENCE'/)
      IF(NTI-39)600,600,601
  601 NTI=39
  600 READ (2,130) (TTAB5(M),M=1,NTI)
```

```
PAGE    3                              NAME       TEST2

      READ (2,130) (TTAB1(M),M=1,NTI)
  130 FORMAT (20X,12F5.3)
      DO 89 I=L1,L2
      L=I+1
      IRXI=I+(I*I-1)/2.0
      READ (1'IRXI) SIRXI
      DO 89 J=L,K
      IRXJ=J+(J*J-J)/2.0
      READ (1'IRXJ) SIRXJ
      IRX=I+(J*J-J)/2.0
      READ (1'IRX) SIRX
      XIJ=SQRT ((SIRXI-2.0*SIRX+SIRXJ)*VE)
      T=(A(I)-A(J))/XIJ
      IF(T)1000,900,900
 1000 T=-T
  900 ITAB=ID(I)-ID(J)
      IF(ITAB)30,32,32
   30 ITAB=-ITAB
   32 IF(ITAB-39)8,6,6
    6 ITAB=39
    8 IF(T-TTAB5(ITAB))18,49,49
   18 LU=1
      CALL DATSW (4,IX)
      GO TO (87,89),IX
   49 II=1
      IF(T-TTAB1(ITAB))19,13,13
   19 LU=2
      GO TO 87
   13 LU=3
   87 WRITE (3,105) I,J,XIJ,T, IDENT (LU)
  105 FORMAT(1X,2I3,5X,F14.4,5X,F7.2,5X,A2)
   89 CONTINUE
      IF(II) 198,198,200
  198 WRITE (3,199)
  199 FORMAT (5X,'NONE OF THE COMPARISONS ARE SIGNIFICANT'/)
  200 WRITE (3,909)
  909 FORMAT('1')
      IJ=ISA
      WRITE (1'IJ) (X(JO),JO=1,NTA)
      CALL LINK (MAIN2)
      END
```

Example number 1:

The data and output listing given below correspond to the analysis of a partially-balanced incomplete-block design with 9 treatments (3 × 3 double lattice), with two replicates, 3 blocks per replicate, and a covariance factor.

DATA:

```
EXAMPLE NO. 1        PBIB
 9 3 2 184 1 12
              1 2 1        56  8128
              1 2 2        51  8138
              1 2 3        63  8231
              1 1 4        63  8279
              1 1 5        60  8170
              1 1 6        63  8251
              1 3 7        62  8253
              1 3 8        75  8386
              1 3 9        71  8244
              2 2 1        55  8111
              2 3 2        63  8249
              2 1 3        60  8185
              2 1 4        58  8173
              2 2 5        58  8066
              2 3 6        66  8264
              2 3 7        62  8252
              2 1 8        62  8228
              2 2 9        71  8249

 3 DF  P=.05          3183 3193 3193 3193 3193 3193 3193 3193
 3 DF  P=.01          5841 5884 5884 5884 5884 5884 5884 5884
```

OUTPUT

```
OBSERVATION EQUATIONS
Y(I,J,K,L) , T(I) , B(J) , R(K) , COV(L)

0.56 2. 0. 0. 0. 0. 0. 0. 0. 0. 1. 0. 0. 1.81.
0.51 0. 1. 0. 0. 0. 0. 0. 0. 0. 1. 0. 0. 1.81.
0.63 0. 0. 1. 0. 0. 0. 0. 0. 0. 1. 0. 0. 1.82.
0.63 0. 0. 0. 1. 0. 0. 0. 0. 1. 0. 0. 0. 1.82.
0.60 0. 0. 0. 0. 1. 0. 0. 0. 1. 0. 0. 0. 1.81.
0.63 0. 0. 0. 0. 0. 1. 0. 0. 1. 0. 0. 0. 1.82.
0.62 0. 0. 0. 0. 0. 0. 1. 0.-1.-1. 0. 0. 1.82.
0.75 0. 0. 0. 0. 0. 0. 0. 1.-1.-1. 0. 0. 1.83.
0.71-1.-1.-1.-1.-1.-1.-1.-1.-1.-1. 0. 0. 1.82.
0.55 1. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 1.-1.81.
0.63 0. 1. 0. 0. 0. 0. 0. 0. 0. 0.-1.-1.-1.82.
0.60 0. 0. 1. 0. 0. 0. 0. 0. 0. 0. 1. 0.-1.81.
0.58 0. 0. 0. 1. 0. 0. 0. 0. 0. 0. 1. 0.-1.81.
0.58 0. 0. 0. 0. 1. 0. 0. 0. 0. 0. 0. 1.-1.80.
0.66 0. 0. 0. 0. 0. 1. 0. 0. 0. 0.-1.-1.-1.82.
0.62 0. 0. 0. 0. 0. 0. 1. 0. 0. 0.-1.-1.-1.82.
0.62 0. 0. 0. 0. 0. 0. 0. 1. 0. 0. 1. 0.-1.82.
0.71-1.-1.-1.-1.-1.-1.-1.-1. 0. 0. 0. 1.-1.82.

MATRIX OF CORRECTED SQUARES AND CROSS PRODUCTS
SAA,SAB,SBB,SAC,SBC,SCC,....SNN

 4.000000   2.000000   4.000000   2.000000   2.000000   4.000000   2.000000   2.000000   2.000000   4.000000
 2.000000   2.000000   2.000000   2.000000   4.000000   2.000000   2.000000   2.000000   2.000000   2.000000
 4.000000   2.000000   2.000000   2.000000   2.000000   2.000000   2.000000   4.000000   2.000000   2.000000
 2.000000   2.000000   2.000000   2.000000   2.000000   4.000000   1.000000   1.000000   1.000000   2.000000
 2.000000   2.000000   1.000000   0.000000   6.000000   2.000000   2.000000   2.000000   1.000000   1.000000
 1.000000   0.000000   0.000000   3.000000   6.000000   0.000000  -1.000000   1.000000   1.000000   0.000000
-1.000000  -1.000000   1.000000   0.000000   0.000000   6.000000   0.000000  -2.000000  -1.000000  -1.000000
 0.000000  -2.000000  -2.000000  -1.000000   0.000000   0.000000   3.000000   6.000000   0.000000   0.000000
 0.000000   0.000000   0.000000   0.000000   0.000000   0.000000   0.000000   0.000000   0.000000   0.000000
18.000000  -2.540000  -1.060000  -0.770000  -0.410000  -2.570000   0.219999   0.119999   1.209999  -1.830000
-3.860000  -1.789999  -3.389999   3.350000   9.462219  -0.310000  -0.260000  -0.190000  -0.210000  -0.240000
-0.130000  -0.179999  -0.049999  -0.220000  -0.380000  -0.110000  -0.070000   0.089999   0.614920   0.059650
-0.059999

MATRIX OF GAUSS MULTIPLIERS
CAA,CAB,CBB,CAC,CBC,CCC, ...CNN

 1.680674   0.500321   0.913530  -0.802696  -0.396053   1.302071  -0.626060  -0.393336   0.363655   0.074794
 1.650477   0.647695  -1.473181  -0.747745   3.353111  -0.014648   0.075007  -0.287003  -0.068871   0.247435
 0.689456  -0.047224   0.058934  -0.261217  -0.220779   0.027747   0.017906   0.680736  -1.951621  -1.047381
 1.412960   0.606669  -3.071990  -0.434262  -0.210254   3.206716  -0.369999  -0.126273   0.491957  -0.004236
-1.005315  -0.294349   0.054439   0.958008   0.672714   0.524502   0.146219  -0.813327  -0.227190   1.326537
 0.223058   0.199069  -1.203345  -0.576516   0.594338   1.158529   0.627918  -1.051356  -0.696752   1.815970
 0.268137   0.234489  -2.065989  -0.496962   0.771326   1.526374  -0.147048   0.146202   0.051603   0.077053
-0.099863   0.122380   0.119965  -0.021216  -0.035607   0.055358  -0.144571   0.450017  -0.253064  -0.124874
 0.200341   0.114659  -0.411939  -0.037941  -0.029811   0.445503   0.120079  -0.186373  -0.261423  -0.018762
 0.116722   1.503471   0.741828  -1.1901-5  -0.691144   2.447166   0.225396   0.177097  -2.046555  -0.713344
 1.107169   1.553009   0.111460  -0.475745   2.229200

EXAMPLE NO. 1       PBIB
```

MATRIX OF REGRESSION COEFFICIENTS
T(I) , B(J) , R(K) , COV(L)

```
 0.0014   -0.0387    0.0022   -0.0108    0.0476   -0.0008   -0.0424   -0.0109   -0.0202   -0.0000
-0.0085    0.0036   -0.0274    0.0737
```

TREATMENT	REP	MEAN	T-COEF	PREDICTION
1	2	0.6550	0.0014	0.6231
2	2	0.5700	-0.0387	0.5828
3	2	0.6150	0.0022	0.6239
4	2	0.6049	-0.0108	0.6107
5	2	0.5899	0.0476	0.6693
6	2	0.6450	-0.0008	0.6207
7	2	0.6200	-0.0424	0.5791
8	2	0.6550	-0.0109	0.6107
9	2	0.7100	0.0525	0.6742

GENERAL MEAN = 0.6216

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES 1/	MEAN SQUARE	F 2/
	8.	0.0416	0.0052	23.74
	4.	0.0144	0.0036	16.54
	1.	0.0004	0.0004	2.05
	1.	0.0024	0.0024	11.15
	3.	0.0006	0.0002	

1/ REDUCTION IN SUM OF SQUARES AS EACH FACTOR IS INTRODUCED
2/ F TEST VALID FOR BALANCED DESIGNS,APPROXIMATE FOR UNBALANCED ONES.

STANDARD DEVIATION = 0.0147
COEFFICIENT OF VARIABILITY = 2.3806

TEST OF SIGNIFICANCE

TREATMENT	PREDICTED MEAN
9	0.6742
5	0.6693
3	0.6239
1	0.6231
6	0.6207
4	0.6107
8	0.6107
2	0.5828
7	0.5791

COMPARISONS BETWEEN TREAT.		STANDARD ERROR OF A DIFFERENCE	T	
2	5	0.0255	3.38	*
2	9	0.0231	3.94	*
4	9	0.0191	3.31	*
7	9	0.0186	5.09	*

Example number 2:

The data and output listing below correspond to the analysis of a latin square design with 4 treatments, 4 columns and 4 rows.[2]

DATA:

```
EXAMPLE NO. 2          LATIN SQUARE
 4 4 4 163 1              1
LAT-SQ       1 1 2       01
LAT-SQ       2 1 1       03
LAT-SQ       3 1 3       15
LAT-SQ       4 1 4       30
LAT-SQ       1 2 1       02
LAT-SQ       2 2 4       07
LAT-SQ       3 2 2       16
LAT-SQ       4 2 3       27
LAT-SQ       1 3 4       03
LAT-SQ       2 3 3       07
LAT-SQ       3 3 1       11
LAT-SQ       4 3 2       23
LAT-SQ       1 4 3       03
LAT-SQ       2 4 2       05
LAT-SQ       3 4 4       14
LAT-SQ       4 4 1       13

 6 DF  P=.05            2447 2536 2580 2602 2612 2614 2614 2614
 6 DF  P=.01            3707 3846 3924 3970 3999 4016 4026 4031
```

OUTPUT:

```
                                   OBSERVATION EQUATIONS
                                   Y(I,J,K,L) , T(I) , B(J) , R(K) , COV(L)

  1.00 0. 1. 0. 1. 0. 0. 1. 0. 0.
  3.00 1. 0. 0. 1. 0. 0. 0. 1. 0.
 15.00 0. 0. 1. 1. 0. 0. 0. 0. 1.
 30.00-1.-1.-1. 1. 0. 0.-1.-1.-1.
  2.00 1. 0. 0. 0. 1. 0. 1. 0. 0.
  7.00-1.-1.-1. 0. 1. 0. 0. 1. 0.
 16.00 0. 1. 0. 0. 1. 0. 0. 0. 1.
 27.00 0. 0. 1. 0. 1. 0.-1.-1.-1.
  3.00-1.-1.-1. 0. 0. 1. 1. 0. 0.
  7.00 0. 0. 1. 0. 0. 1. 0. 1. 0.
 11.00 1. 0. 0. 0. 0. 1. 0. 0. 1.
 23.00 0. 1. 0. 0. 0. 1.-1.-1.-1.
  3.00 0. 0. 1.-1.-1.-1. 1. 0. 0.
  5.00 0. 1. 0.-1.-1.-1. 0. 1. 0.
 14.00-1.-1.-1.-1.-1.-1. 0. 0. 1.
 13.00 1. 0. 0.-1.-1.-1.-1.-1.-1.

                                   MATRIX OF CORRECTED SQUARES AND CROSS PRODUCTS
                                   SAA,SAB,SBB,SAC,SBC,SCC,....SNN

 8.000000   4.000000   8.000000   4.000000   4.000000   8.000000   0.000000   0.000000   0.000000   8.000000
 0.000000   0.000000   0.000000   4.000000   8.000000   0.000000   0.000000   0.000000   4.000000   4.000000
 8.000000   0.000000   0.000000   0.000000   0.000000   0.000000   0.000000   8.000000   0.000000   0.000000
 0.000000   0.000000   0.000000   0.000000   4.000000   8.000000   0.000000   0.000000   0.000000   0.000000
 0.000000   0.000000   4.000000   4.000000   8.000000 -25.000000  -9.000000  -2.000000  14.000000  17.000000
 9.000000 -84.000000 -71.000000 -37.0000001255.000000

                                   MATRIX OF GAUSS MULTIPLIERS
                                   CAA,CAB,CBB,CAC,CBC,CCC, ...CNN

 0.187500  -0.062500   0.187500  -0.062500  -0.062500   0.187500   0.000000   0.000000   0.000000   0.187500
 0.000000   0.000000   0.000000  -0.062500   0.187500   0.000000   0.000000   0.000000  -0.062500  -0.062500
 0.187500   0.000000   0.000000   0.000000   0.000000   0.000000   0.000000   0.187500   0.000000   0.000000
 0.000000   0.000000   0.000000   0.000000  -0.062500   0.187500   0.000000   0.000000   0.000000   0.000000
 0.000000   0.000000  -0.062500  -0.062500   0.187500

EXAMPLE NO. 2       LATIN SQUARE

                                   MATRIX OF REGRESSION COEFFICIENTS
                                   T(I) , B(J) , R(K) , COV(L)

 -4.0000   -0.0000    1.7500    0.9999    1.7500   -0.2500   -9.0000   -5.7500    2.7500

TREATMENT  REP        MEAN       T-COEF      PREDICTION

    1       4        7.2500     -4.0000       7.2500
    2       4       11.2500     -0.0000      11.2500
    3       4       13.0000      1.7500      13.0000
    4       4       13.5000      2.2500      13.5000
```

GENERAL MEAN = 11.2500

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES 1/	MEAN SQUARE	F 2/
	3.	96.5000	32.1666	3.54
	3.	41.5000	13.8333	1.52
	3.	1062.5000	354.1666	38.99
	6.	54.4999	9.0833	

1/ REDUCTION IN SUM OF SQUARES AS EACH FACTOR IS INTRODUCED
2/ F TEST VALID FOR BALANCED DESIGNS,APPROXIMATE FOR UNBALANCED ONES.

STANDARD DEVIATION = 3.0138
COEFFICIENT OF VARIABILITY = 26.7898

TEST OF SIGNIFICANCE

TREATMENT	PREDICTED MEAN
4	13.5000
3	13.0000
2	11.2500
1	7.2500

COMPARISONS BETWEEN TREAT.	STANDARD ERROR OF A DIFFERENCE	T	
1 3	2.1311	2.69	*
1 4	2.1311	2.93	*

Mariano Antoni
B. G. Capó
Statistics Section