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Using the Bernoulli model to analyze the distribution of course withdrawals at UPR-Bayamón

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ABSTRACT

Using a rich panel data comprising 39,337 courses offered in the UPR-Bayamón during forty-one consecutive terms, this paper analyses the distribution of course withdrawals, estimating four parameters per course: the proportion of withdrawals and its variance, as well as the coefficients of skewness and kurtosis. Evidence suggests that the characteristics of courses, students, and, particularly, unobservable faculty heterogeneity exert a strong and statistically significant effect on these parameters over time and within academic fields. Faculty members and students engage in a *shopping-around* process where both parties improve their well-being at the expense of the academic standards and the quality of the education provided.

Keywords: course withdrawals, Bernoulli model, moment-generating function, skewness, kurtosis

Uso del modelo de Bernoulli para analizar la distribución de bajas parciales por curso en la UPR-Bayamón

RESUMEN

Usando un archivo longitudinal de los 39,337 cursos ofrecidos en la UPR-Bayamón durante 41 semestres consecutivos, se analiza la distribución de bajas parciales por curso a través de los primeros cuatro momentos: media, varianza, asimetría, y curtosis. Las características de los cursos, de los estudiantes, y muy particularmente, la heterogeneidad inobservable de los profesores, ejercen una fuerte y significativa influencia sobre el comportamiento de los momentos a través del tiempo. Parecería, que profesores y estudiantes están involucrados en un proceso de *ir de compras* que les beneficia mutuamente a expensas de los estándares académicos y de la calidad de la educación provista.

Palabras clave: bajas parciales por curso, modelo de Bernoulli, función generatriz de momentos, asimetrías, curtosis

Introduction

One of the most significant challenges that institutions of higher education face is the establishment of a selective admission policy which allows them to identify and admit, from a total pool of applicants, the candidates most able and likely to academically succeed: the greater the institutional economic shortage, the greater the urgency. Suppose that, to reach such an objective, the university administrators consider designing an ideal standardized test. According to Rothstein (2004), the *ideal test* should be able to predict as accurately as possible which applicants would be most successful if admitted. That is, all applicants whose performance in the test exceeds a determinate threshold after admission would be likely to (a) succeed academically and (b) fulfill all academic requisites during the allotted time. Should this be the case, the institutional admission policy would be quite simple and efficient. However, designing and implementing such a test is extremely difficult, if not impossible. Many diverse factors influence student academic achievement, which are difficult to identify and measure, and most are beyond student and institutional control.

Since the academic year 1979-80, the University of Puerto Rico (UPR) —the country's public university system composed of eleven campuses across the Island— has adopted as its official admission policy a standardized test administered by the Puerto Rico (PR) and Latin America Office of the College Entrance Examination Board (CEEB), named the General Admission Index (GAI).[1] Every

[1] The GAI is the weighted mean of the high school GPA (HS-GPA) (50%) and the scores in the verbal aptitude (25%) and mathematics aptitude (25%) sections of the CEEB test.

year, each of the UPR's eleven campuses establishes the minimum GAI required by its different academic programs in response to trends in enrollment demand and the program's capacity.[2] The fact that the GAI required for each program is made public every year has led from its inception to a self-inclusion/exclusion process by which students themselves decide whether to apply to the UPR (and to a particular program), based on their GAI and the minimum established by the program. Students' admissions to each academic program follow a strict order entirely defined by their GAI. It is expected that these students will be able to sort themselves in such a way that more (less) academically able are admitted to the highest (lowest) selective programs with more (less) inherent difficult content.

Thus, the role of the *ideal test* described earlier has been ascribed to the GAI. The issue to be settled is whether the GAI satisfies conditions (a) and (b) previously mentioned. Of course, the answer is no. Because of the inaccuracies of GAI, there are several endemic academic problems whose incidence varies among and within the eleven UPR campuses. For instance, to be admitted to the UPR at Bayamón (UPR-Bayamón), many students whose GAI is below the minimum apply to non-desired programs for which they qualify, looking for an eventual possible transfer to their desired program. The strategy consists of enrolling in courses in their desired program even though they are officially admitted to a different one. Because the academic requirements and contents of the programs could differ significantly, the likelihood of course failures and withdrawals increases. Moreover, such a strategy lengthens time until graduation, increasing the opportunity cost of schooling. Eventually, many such students withdraw from the institution because of failure to be admitted to their desired programs. Furthermore, it should be mentioned that

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[2] For instance, by the academic year 2015-16, at the UPR-Bayamón, the minimum GAI required to be admitted to the bachelor programs of Accounting, Biology and Natural Sciences, as well as Mechanical Engineering transfer program, was 280, 305, 315 and 335, respectively. For programs such as Engineering Technologies, Physical Education and Education the minimum GAI was 240, 270 and 270, respectively.

these endemic problems are also prevalent even among students who were admitted to their desired programs from the beginning. Therefore, the official admission policy generates undesirable by-products like academic failures, too many applications for program transfers, as well as total and partial withdrawals. Among these problems, this study seeks to analyze the distribution of withdrawals (*W*) observed in the 39,337 courses offered during 41 consecutive terms (including summer sessions) at UPR-Bayamón from fall 1995-96 to fall 2015-16. To the best of my knowledge, the extant literature lacks studies devoted to analyzing the implications and consequences of the proportion of course withdrawals on the education process; this paper seeks to fill such a gap.

To this end, this paper adopts the Bernoulli probability model and derives the moment-generating functions around its origin and mean. For each course, the objective is to calculate the following four parameters: first, the proportion of withdrawals, which equals the first moment around the origin; second, the variance of its distribution, which is equal to the second moment around the mean; third, the coefficient of skewness, using the third moment around the mean; and fourth, the coefficient of kurtosis, using the fourth moment around the mean. Using different econometric specifications, including random- and fixed-effects models, allows the modeling of these four parameters. This paper uses a rich and detailed panel data containing time-varying variables describing faculty, student, and course characteristics to fulfill this objective.

This study contributes to the extant literature by (a) being the first to analyze in detail the distribution of withdrawals and its key moments at the course level, (b) using a rich panel data comprising all courses offered at UPR-Bayamón during 41 consecutive terms, (c) including time-varying variables describing in detail the faculty characteristics, and showing that courses, faculty and students characteristics exert strong and significant effects on the estimated models, and (d) presenting empirical evidence pointing to the conclusion that faculty and students engage in a symbiotic relationship where both parties improve their well-being at the expense of diminishing academic standards.

The remainder of the paper is organized as follows. Section 2 justifies adopting the Bernoulli model based on its simplicity and statistical properties. Section 3 describes the nature of the data and the specification of the statistical models to be estimated. Section 4 discusses the empirical results, as well as their policy implications. Finally, Section 5 closes the study with a summary and conclusions.

Motivating the Adoption of the Bernoulli Probability Model

When and why do students decide to withdraw from a course? Although the answer to this question has dramatic policy implications for students and universities since withdrawals entail significant cost consequences, the research published on this topic is limited. Wollman and Lawrence (1984), Adams and Becker (1990), Dunwoody and Frank (1995), and Miller (1997) constitute notable exceptions. However, inspired by the original work of Spady (1970, 1971) and Tinto (1975), there is an extensive and diverse body of published research related to the withdrawals of students from college. This is the first research to analyze the determinants of the distribution of withdrawals and their key moments at the course level.

The study of the distribution of course withdrawals among and within academic programs and across time is relevant for several reasons. Withdrawals can increase student time until graduation and the total cost of the degree. Moreover, they could predict or signal total withdrawals and attrition from college, decreasing college graduation rates. At one time, Dunwoody and Frank (1995) raised the issue that individual course withdrawal could have the highest impact on overall retention, attrition, and institutional success. For some researchers (e.g., Zwick & Sklar, 2005), the best criterion to measure an institution's academic success is, precisely, the proportion of students who complete their degrees in the allotted time. In this context, low graduation rates negatively impact institutional rankings and, consequently, their ability to attract students with more significant academic potential. Moreover, student attrition represents a fiscal cost to institutions in terms of lost revenues from tuition, room and board, and alum donations (Raisman, 2013; Schuh, 2005). Attrition also constitutes a problem for society in general by reducing the availability of college-educated workers in the labor market (Bound et al., 2007). It also negatively impacts lower tax receipts for federal and state governments (Schneider & Yin, 2011). Although these considerations are beyond the scope of this research, they illustrate how important it is to model the determinants of the distribution of withdrawals at the course level.

From Adams and Becker (1990), it will be hypothesized that students want to maximize their utility function subject to the constraints imposed by their economic and academic environment. Students derive utility from their present and future stream of consumption of the goods and services they will be able to buy in the market as a product of their investment in human capital through education. Education is costly in terms of money and opportunity cost. Therefore, withdrawing from courses would entail a waste of money and increase the opportunity cost of schooling by lengthening the time until graduation. It seems reasonable to posit that the disutility derived by students directly varies with the intrinsic course difficulty level. However, such a concept is relative and unobservable. Thus, it will be assumed that a student will withdraw whenever the disutility (dissatisfaction) derived from the course is greater than the disutility induced by the cost of withdrawing it.[3] While student disutility or dissatisfaction is not observable, their actions of withdrawing or remaining in the course are. The indicator variable defined in (5) allows us to consider these actions.

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[3] In such a decision-making process, the five top reasons students give for withdrawing from courses (Dunwoody & Frank, 1995) can be considered aggravating circumstances. These top reasons are: (a) I was not happy with my grade, (b) I did not understand the material, (c) I did not like the course, (d) I did not like the professor, and (e) the subject did not interest me.

Usually, the determination of the number of total courses offered by academic fields (AFs) and the number of students enrolled in each one occurs at the beginning of each term. Likewise, by the end of the term, each academic department head knows with certainty the number of students who withdraw from each course and those who remain in it. Suppose each academic unit adopts a coding system such that code "1" represents the students who withdraw and code "0" represents those who do not withdraw. Thus, expression (1) defines the random variable *W*.

$$(1) \quad W = \begin{cases} 1, \text{ if student } i \text{ withdraws from course } j \\ 0, \text{ otherwise} \end{cases}$$

Let N_j and W_j be the total enrollment and the number of students who withdraw from course *j* after the deadline to add or drop a course, respectively.[4] For this study, the outcomes "success" and "failures" represent students who withdraw and those who do not withdraw from a course, respectively.[5] Thus, (2) defines the proportion of withdrawals observed in course *j*, which is the same as its relative frequency.

$$(2) \quad \pi_j = \frac{W_j}{N_j}$$

Given that the purpose of this study is to model π observed in each course offered at UPR-Bayamón over 41 consecutive terms, the selection of the accurate probability model is of crucial impor-

[4] A student who partially or totally withdraws prior to the last day to drop/add courses will be refunded 100% of the tuition paid. If the student withdraws after the deadline of the drop/add period but before certain established period (typically 8-11 days after it) will be refunded 50% of the tuition paid. After such a period, there will be no refund. The deadline for total withdrawal is the last day of classes, while the deadline for partial withdrawal is approximately two weeks before the last day of classes.

[5] In statistics terminology, a favorable or successful outcome does not necessarily imply an outcome that is desirable in practice (Chow, 1989). Whenever the outcome we are interested in occurs, it is classified as a success; otherwise, it is classified as a failure.

tance. The Bernoulli model describes the behavior of a random discrete variable that takes on only two values, arbitrarily designated as 0 (failure) or 1 (success). The probability of (1) is equal to the proportion of success in the universe, while the probability of (0) is its complement.

$$(3) \quad P(W=1)=f(1)=\pi_j=\frac{W_j}{N_j}\Rightarrow f(0)=\frac{N_j-W_j}{N_j}=1-\pi_j$$

Therefore, (4) defines the probability mass function of a Bernoulli random variable.

$$(4) \quad P(W=w)=f(w)=\begin{cases}\pi^w(1-\pi)^{1-w}, & \text{if } w=0 \text{ or } w=1\\ 0, & \text{otherwise}\end{cases}$$

Following the nomenclature adopted by Rice (1995), if W constitutes the event that student i withdraws from course j, then the indicator random variable I_w takes on the value 1 if W occurs and the value 0 if W does not occur. Hence, the indicator I_w is a Bernoulli random variable.

$$(5) \quad I_W(w)=\begin{cases}1, & \text{if } w\in W\\ 0, & \text{otherwise}\end{cases}$$

It follows that each course offered at UPR-Bayamón analyzed in this study constitutes a unique and nonreplicable Bernoulli academic experiment whose results can be classified into two mutually exclusive and collectively exhaustive outcomes: failure (0) or success (1); with probabilities equal to $(1-\pi)$ and π, respectively. Expression (6) defines the expected value $(E(W))$ and the variance $(\sigma^2(W))$ of the Bernoulli experiment conducted in course j.

$$(6) \quad E(W)=1\cdot\pi+0\cdot(1-\pi)=\pi_j=\frac{W_j}{N_j} \text{ and } \sigma^2(W)=\pi_j(1-\pi_j)$$

Therefore, modeling the proportion of withdrawals (π_j) observed in course *j* is identical to model a Bernoulli variable's probability of success (withdrawing from course *j*). The advantage of such an approach resides in the fact that the Bernoulli model is entirely determined by π, its single parameter.

The Moment-Generating Functions: Interesting Analytical Results

The first moment around the origin is equal to π. This calculation requires taking the first derivative with respect to *t* (evaluated at $t = 0$) from the corresponding moment-generating function.[6] The superior moments around the origin are all equal to π. However, moments around the mean are more interesting. Let μ_1, μ_2, μ_3, and μ_4 be the first four moments around the mean. To find them, it is necessary to take the first, second, third, and fourth derivatives with respect to *t* and to evaluate each one at $t = 0$; Table 1 reports their values, as well as the coefficients of skewness (C_S) and kurtosis (C_K). The units of measurement of μ_3 and μ_4 influence their respective size. Therefore, considered alone, they are poor measures of skewness and kurtosis, respectively. Such dimensionality disappears, defining each coefficient as a relative measure, as done in (7).[7]

$$(7)\quad \begin{cases} C_S = \dfrac{\mu_3}{\left[\sigma(W)\right]^3} \\ C_K = \dfrac{\mu_4}{\left[\sigma(W)\right]^4} \end{cases}$$

As expected, $\mu_1 = 0$. The second moment is equal to the variance. It is an open downward parabola, which reaches its absolute maximum (0.25) at $\pi = 0.5$. Obviously, it is zero in the extremes, at

[6] The respective moment-generating functions around the origin and the mean are: $m_W(t) = (1 - \pi) + \pi e^t$ and $m_W(t) = (1 - \pi)e^{-\pi t} + \pi e^{(1-\pi)t}$.

[7] For details, refer to Chow (1989).

Table 1

Key Parameters of the Bernoulli Probability Model

1. $\mu_1' = 0$

2. $\mu_2 = \pi(1-\pi) = \sigma^2(W) \Rightarrow \frac{d\mu_2}{d\pi} = (1-2\pi) = 0 \Rightarrow \pi = 0.5$

3. $\mu_3 = \pi(1-\pi)(1-2\pi) \Rightarrow C_S = \frac{\mu_3}{\left[\sigma(W)\right]^3} = \frac{1-2\pi}{\sqrt{\pi(1-\pi)}} \Rightarrow \begin{cases} C_S > 0, \text{ if } \pi \in (0;\ 0.5) \\ C_S = 0, \text{ if } \pi = 0.5 \\ C_S < 0, \text{ if } \pi \in (0.5;\ 1) \end{cases}$

4. Coefficient of skewness (C_S): if $\begin{cases} C_S = \pm 1 \Rightarrow \text{ the distribution is highly skewed} \\ 0.5 < \left|C_S\right| < 1 \Rightarrow \text{ the distribution is moderately skewed} \\ 0 < \left|C_S\right| < 0.5 \Rightarrow \text{ the distribution is nearly symmetric} \end{cases}$

$$\frac{dC_S}{d\pi} = C_S' = \frac{-1}{2\left[\pi(1-\pi)\right]^3} < 0 \ \forall \ \pi \in (0;\ 1)$$

$$\frac{d^2C_S}{d\pi^2} = C_S'' = \frac{3(1-2\pi)}{4\left[\pi(1-\pi)\right]^5} \Rightarrow \begin{cases} C_S'' > 0, \text{ if } \pi \in (0;\ 0.5) \\ C_S'' = 0, \text{ if } \pi = 0.5 \\ C_S'' < 0, \text{ if } \pi \in (0.5;\ 1) \end{cases}$$

5. $\mu_4 = \pi(1-\pi)\left[1-3\pi(1-\pi)\right] \Rightarrow C_K = \mu_4 / \left[\sigma(W)\right]^4 = -3 + \frac{1}{\pi(1-\pi)}$

6. Coefficient of kurtosis (C_K): if $\begin{cases} C_K = 3 \Rightarrow \text{ the distribution is said to be } \textit{mesokurtic} \\ C_K < 3 \Rightarrow \text{ the distribution is classified as } \textit{platykurtic} \\ C_K > 3 \Rightarrow \text{ the distribution is said to be } \textit{leptokurtic} \end{cases}$

$$\frac{dC_K}{d\pi} = C_K' = \frac{-1+2\pi}{\left[\pi(1-\pi)\right]^2} \Rightarrow \begin{cases} C_K' < 0, \text{ if } \pi \in (0;\ 0.5) \\ C_K' = 0, \text{ if } \pi = 0.5 \\ C_K' > 0, \text{ if } \pi \in (0.5;\ 1) \end{cases}$$

$$\frac{d^2C_K}{d\pi^2} = C_K'' = \frac{2\left[1-3\pi(1-\pi)\right]}{\left[\pi(1-\pi)\right]^3} > 0 \ \forall \ \pi \in (0;\ 1)$$

$\pi = 0$ or $\pi = 1$. The third moment (μ_3) measures skewness because it preserves the sign of the deviance with respect to the mean. The coefficient of skewness (C_S) is undefined either at $\pi = 0$ $(\rightarrow -\infty)$

or at $\pi = 1$ ($\rightarrow -\infty$). It is a positive decreasing convex function ($C''_S > 0$) on $\pi \in (0; 0.5)$, reaches its inflection point at $\pi = 0.5$, and continues decreasing as a negative concave function ($C''_S < 0$) on $\pi \in (0.5; 1)$. The coefficient of kurtosis (C_K) is a positive U-shaped convex function symmetric with respect to the line $\pi = 0.5$. The coefficient, as well as its first and second derivatives are undefined at points $\pi = 0$ and $\pi = 1$. The coefficient decreases on $\pi \in (0; 0.5)$, reaches its absolute minimum equal to 1 on $\pi = 0.5$, and increases unbounded on $\pi \in (0.5; 1)$.

As mentioned earlier, π determines, completely and uniquely, the four parameters of interest for this study. Three different points are of key interest in the range of π: $\pi = 0$, $\pi = 0.5$ and $\pi = 1$. There are 11,206 (28.49%) courses where $\pi = 0$, and none where $\pi = 1$ (see Section 4). For all courses where $\pi = 0$, $\sigma^2(W) = 0$; however, C_S and C_K are undefined. In all courses where $\pi = 0.5$, $C_S = 0$, implying a symmetric distribution of withdrawals, while $\sigma^2(W)$ and C_K reach their maximum and minimum values (0.25 and 1, respectively). On the other hand, the distribution of withdrawals is skewed to the left in all courses, where $\pi > 0.5$, since $C_S < 0$.

Thus, once π_j is known, it is straightforward to characterize the degrees of skewness and kurtosis prevailing in course j according to these analytical results. Moreover, the mean and variance of C_S and C_K distributed by specific courses or AFs can be computed and depicted over the 41 terms considered in this study for analytical comparison. Therefore, the simplicity of the Bernoulli model allows us to easily analyze the distribution of course withdrawals looking for policy measures that improve the academic process.

Data Description

The UPR-Bayamón is an autonomous unit of the UPR system. Accredited by the Middle States Association of Colleges and Secondary Schools, it offers associate and bachelor's degrees, as well as articulated transfer programs to the Río Piedras, Mayagüez, and Medical Sciences campuses. In the fall of 2024, total enrollment at UPR-Bayamón was 2,852, including 2,520 full-time students.

For each one of the 39,337 courses offered in UPR-Bayamón from 1995 to 2015, the following variables are available: enrollment; instructor who taught the course; letter grade distribution (*As, Bs, Cs, Ds, Fs,* and *W*); GPA; variance of the GPA, and AFs (21 dummies). As proxies to account for student quality at the course level, this research uses the mean and variance of the following variables: high school graduation GPA (HS-GPA), GAI, and the score on each of the five sections of the CEEB.[8] Furthermore, the proportions of students by gender and type of high school (public or private) are available for each offered course. Dummies control for academic schedules (weekdays and hours) and for summer terms. For each faculty member in the sample, the following time-varying variables are available: age, rank, degree, and tenure status. Dummies account for the instructor's gender and the presence of courses subject to student evaluations of teaching (SET). The inclusion of a set of forty-one dummies, identifying term/year, allows for capturing time effects.[9] Table 2 describes the variables used.

Models for Estimation

The preceding discussion suggests that the model specified in (8) is appropriate to estimate the equations that predict the proportion of withdrawals and the distribution variance, as well as the coefficients of skewness and kurtosis observed in course *j*, taught by professor *f*. The matrices $\mathbf{X}_j$, $\mathbf{Z}_f$, and $\mathbf{M}_s$, consist of course (*j*), faculty (*f*), and student's (*s*) characteristics, respectively. The vectors β, Φ, and ψ represent parameters to be estimated and ε_{jf} is the composite error term. The inclusion of random- and fixed-effects models allows to account for unobservable faculty heterogeneity (UFH = γ_f).

$$(8)\quad Y_{jf} = \Upsilon_0 + \Upsilon_f + \mathbf{X}_j^T\beta + \mathbf{Z}_f^T\Phi + \mathbf{M}_s^T\psi + \varepsilon_{jf}$$

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8 The CEEB test includes five sections: verbal and mathematical aptitude, and achievement in Spanish, English, and mathematics.

9 Fall sessions include courses offered during summers.

Table 2

Sample Statistics

Dummy variables					
Variable	Mean	Variable	Mean	Variable	Mean
Accounting	0.0539	Marketing	0.0125	Probation	0.0854
	(0.2259)		(0.111)		(0.2794)
Biology	0.0515	Materials Manage-	0.0075	Tenure	0.6155
	(0.221)	ment	(0.0863)		(0.4865)
Chemistry	0.0349	Mathematics	0.1301	Class size 1	0.0598
	(0.1836)		(0.3364)		(0.2372)
Computer	0.0633	Physical Education	0.0375	Class size 3	0.2773
Sciences	(0.2436)		(0.19)		(0.4477)
Economics &	0.0277	Physics	0.0294	Morning	0.541
Statistics	(0.164)		(0.1689)		(0.4983)
Education	0.0595	Office Systems	0.0381	Night	0.0927
	(0.2366)		(0.1914)		(0.29)
Electronic	0.0587	Social Sciences	0.0615	Summer	0.0156
	(0.2351)		(0.2403)		(0.1241)
Engineering	0.0285	Spanish	0.0682	SET	0.1465
Technologies	(0.1663)		(0.2521)		(0.3536)
Engineering	0.01	Female	0.5285	Monday to	0.017
Transfers	(0.0994)	faculty	(0.4992)	Friday	(0.1292)
English	0.0994	Doctorate	0.2545	Monday &	0.3181
	(0.2992)		(0.4356)	Wednesday	(0.4658)
Finance	0.0183	Assistant	0.2256	Tuesday &	0.3671
	(0.1341)		(0.418)	Thursday	(0.482)
Humanities	0.0758	Associate	0.2149	M/T/W	0.1625
	(0.2647)		(0.4107)		(0.3689)
Management	0.0337	Professor	0.2269	M/T/W/Th	0.01
	(0.1804)		(0.4188)		(0.0777)

Table 2 (continued)

Semi-continuous variables					
Variable	Description	*M*	*SD*	Min	Max
π	Proportion of *W*	10.89	12.72	0	92.86
σ^2 (*W*)	Variance of *W*	7.89	7.47	0	25
C_S	Coefficient of skewness	2.68	1.52	-3.33	7.88
C_K	Coefficient of kurtosis	10.5	8.78	1	63.02
Age	Professor's age (years)	47.63	9.64	23	76
GAI	General Application Index	285	23.03	174	372
GAI Var	GAI distribution variance	795.2	686	0	20,031
FSP	Female students' proportion	0.5329	0.2662	0	1
PHSP	Private HS proportion	0.4722	0.1622	0	1

Note: For all dummies, standard deviations are in parentheses, Max. = 1 and Min. = 0.

Results and Discussion

Stylized Facts of the Parameters Across Time

Table 3 reports the mean values of the four parameters computed over the 41 terms analyzed in this study distributed by AFs: π, σ^2 (W) C_S, and C_K. AFs are ordered using the values of π, from smallest to largest, as reference. The parameters π and $\sigma^2(W)$ move in the same direction until $\pi = 0.5$, and the former is always greater than the latter. The values of π are not randomly distributed by AFs. For instance, for all courses offered by the Marketing and Physical Education departments, the respective figures are the lowest: 2.69% and 4.6%. However, for the Economics & Statistics and Mathematics courses, the figures are the highest: 18.31% and 29.22%, respectively. To the extent that π values were directly related to the inherent difficulty of the course, evidence points to the conclusion that Chemistry (14.93%), Economics & Statistics (18.31%), and Mathematics (29.22%) are the most challenging courses. Likewise, Marketing (2.69%), Physical Education (4.6%), and Management (5.42%) are the easiest ones.

A cautionary note is in order here. Higher π values would signal increased course inherent difficulty levels to the extent that academic standards do not decrease over time. According to the leniency hypothesis (Gump, 2007), faculty members can buy higher SET ratings, recruit more students, improve their teaching schedules, or even become more popular by relaxing their academic standards through leniency grading. If so, GPA will increase (implying grade inflation), and withdrawals will decrease among and within courses across time. To test for such a conjecture, all the econometric models include a dummy variable that takes on the value 1 if SET were conducted in the course and 0 otherwise.

The coefficients of skewness and kurtosis move in opposite directions to the course's inherent difficulty level. Marketing exhibits the highest figures: $C_S = 4.24$ and $C_K = 20.14$; while Mathematics exhibits the lowest: 1.21 and 4.08, respectively. Over the period covered in the study, on average, all AFs exhibit skewed

Table 3

Parameters of the Distribution of Withdrawals by AFs

AFs	π	$\sigma^2(W)$	C_S	C_K
Marketing	2.69 (738)	2.49 (738)	4.24 (351)	20.14 (351)
Physical Education	4.6 (1,970)	4.04 (1,970)	3.41 (1,055)	14.01 (1,055)
Spanish	5.27 (2,948)	4.58 (2,948)	3.67 (1,919)	16.16 (1,919)
Management	5.42 (1,423)	4.72 (1,423)	3.67 (947)	16.3 (947)
Social Sciences	5.98 (2,551)	5.15 (2,551)	3.5 (1,731)	15.09 (1,731)
Education	6.18 (2,714)	5.23 (2,714)	3.22 (1,674)	13.04 (1,674)
Finance	6.72 (791)	5.67 (791)	3.21 (515)	13.22 (515)
Humanities	7.42 (2,903)	6.2 (2,903)	3.3 (2,132)	13.95 (2,132)
English	7.56 (3,922)	6.34 (3,922)	3.12 (2,787)	12.46 (2,787)
Materials Management	8.0 (309)	6.55 (309)	2.96 (211)	11.56 (211)
Office Systems	8.09 (1,780)	6.56 (1,780)	2.54 (1,071)	8.59 (1,071)
Engineering Technologies	10.66 (1,203)	8.16 (1,203)	2.29 (801)	7.59 (801)
Physics	10.79 (1,321)	7.8 (1,321)	2.31 (827)	8.24 (827)
Engineering Transfers	10.88 (405)	8.28 (405)	2.38 (281)	8.16 (281)
Computer Sciences	11.22 (2,496)	8.57 (2,496)	2.38 (1,782)	8.25 (1,782)
Biology	11.43 (1,806)	8.91 (1,806)	2.62 (1,449)	9.78 (1,449)
Electronics	12.16 (2,471)	8.89 (2,471)	2.09 (1,652)	6.81 (1,652)
Accounting	14.83 (1,843)	10.78 (1,843)	2.21 (1,517)	7.8 (1,517)
Chemistry	14.93 (1,084)	11.49 (1,084)	2.26 (982)	7.73 (982)
Economics & Statistics	18.31 (875)	12.21 (875)	2.12 (778)	8.14 (778)
Mathematics	29.22 (3,784)	17.5 (3,784)	1.21 (3,659)	4.08 (3,659)
All AFs	10.89 (39,337)	7.89 (39,337)	2.68 (28,131)	10.5 (28,131)

Note. The values of π and $\sigma^2(W)$ are multiplied by 100. Total courses are in parentheses.

to the right and *leptokurtic* distributions. However, there are 194 courses where $0.49 \leq \pi \leq 0.51$. For all of them, $\sigma^2(W)$ reaches its absolute maximum (0.25). Furthermore, C_S tends to zero, implying symmetric distributions; while C_K tends to one, implying *platykurtic* distributions. A total of 106 ($\approx$ 55%) of those courses belongs to Mathematics. Thus, to the extent that π tends to zero, C_S and C_K increase unbounded implying distributions exhibiting higher skewed to the right and greater *leptokurtosis* degrees, respectively. Finally, there are 686 courses where $\pi > 0.5$; out of this number, 465 ($\approx$ 68%)belong to Mathematics. On the other hand, there are 11,206 courses where $\pi = 0$, but only 125 (1%) belong to Mathematics. Thus, according to student withdrawal decisions, Mathematics courses are the most difficult, independent of the criteria used to measure difficulty.

Figures 1 and 2 clearly depict the growth path of the student quality proxies and the four parameters under study. Four proxies account for student quality at the course level: GAI (see footnote 1), HS-GPA, mathematics, and verbal aptitude. Although GAI tends to increase over time, it should be mentioned that such a tendency is pushed by the self-sustained growth path of HS-GPA, which tends to increase over time (implying grade inflation). However, according to mathematics and verbal aptitude figures, student quality decreases over time. It should be mentioned that this decreasing tendency is consistent with empirical evidence documented at the international level, particularly evidence from Norway.[10] Given that, on average, students are academically less able each term, two results should be expected by course: diminishing GPA and increasing π.

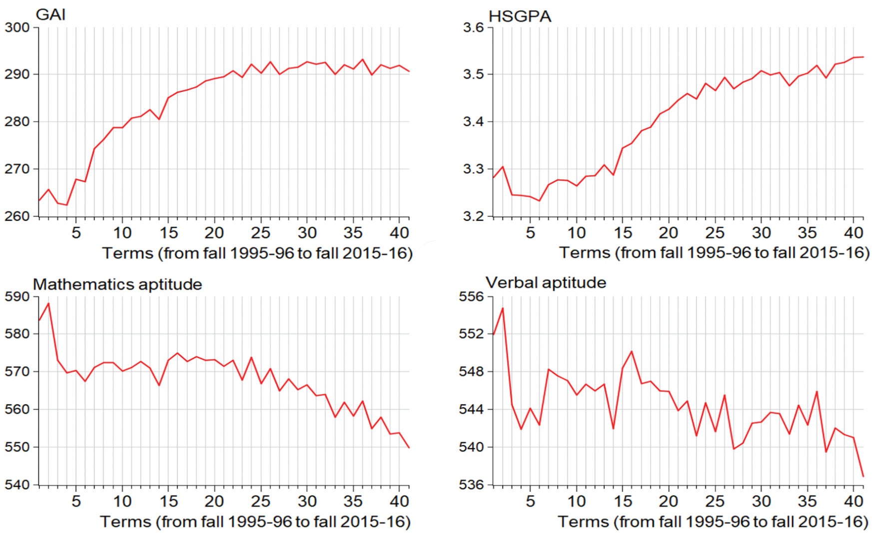
The last two columns of Table 4 report the GPAs for the full sample (39,337 courses), as well as the GPAs for the subsample where $W \geq 1$ (28,131 courses) distributed by terms. Each GPA in the first column is greater than its counterpart in the second one, and both series increase over time, implying grade inflation

[10] Refer to Bratsberg and Rogeberg (2018), and the references cited therein. Professor Bratsberg kindly brought to my attention and provided me a copy of his paper. For his generosity, I am grateful.

since simultaneously, student quality is diminishing. On the other hand, during the forty-one terms studied, the overall π is 10.89%. As shown in the sixth column of Table 4 and clearly depicted in the first graph of Figure 1, π decreased over time, from 13.21% in the fall 1995 term to 9.88% in the fall 2015 term. Therefore, contrary to what should be expected, evidence points to increasing GPAs and decreasing π.[11]

Figure 1

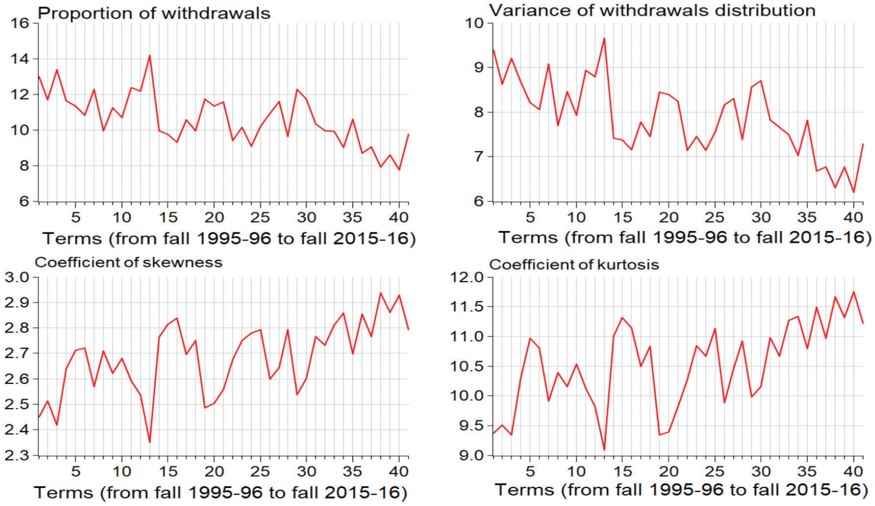
Students Quality Proxies' Growth-Path



[11] These results are consistent with an academic environment characterized by diminishing standards and grade inflation. Several recent studies conducted in the institution have documented such a problematic. For details, refer to Matos-Díaz (2012, 2014, 2018) and Matos-Díaz & García-Vázquez (2014).

Figure 2

Growth-Path of the Key Parameters of the Distribution of Withdrawals



It should be emphasized that this inverse relationship between GPA and π documented across time is a robust one observed among and within AFs. Table 5 reports the evidence. Once again, AFs are ordered using the values of π, from smallest to largest, as reference. Conversely, the respective GPAs reported in the three columns run from largest to smallest. Moreover, for each academic field, the GPAs observed in courses where $W \geq 1$ is lower than the respective one observed in the full sample, where $W \geq 0$, which in turn is lower than the one observed in courses where $W = 0$. Therefore, either over time or between and within AFs, GPA and π move in opposite directions. This result is at odds with that reported in the extant literature (Matos-Díaz, 2018).

Figure 2 also depicts the growth-paths of σ^2 (W) C_S, and C_K. Like π, σ^2 (W) decreases over time. Conversely, C_S and C_K exhibit an increasing tendency over time. Thus, the distributions of course withdrawals become more skewed to the right and more *leptokurtic*, implying greater academic homogeneity among and within courses over time.

Table 4

Stylized Facts of Withdrawals by Terms

Academic Year	Enrollment (A)	Courses (B)	C. Size (A/B)	W (C)	% W (C/A)	GPA ($W > 0$)	GPA ($W = 0$)
95/96: F	21,534	914	24	2,845	13.21	2.54	2.42
95/96: S	19,720	869	23	2,348	11.91	2.66	2.51
96/97: F	25,010	1037	24	3,440	13.75	2.51	2.36
96/97: S	21,948	902	24	2,611	11.9	2.64	2.5
97/98: F	25,193	1024	25	2,936	11.65	2.53	2.4
97/98: S	23,906	1010	24	2,601	10.88	2.62	2.48
98/99: F	25,541	1040	25	3,167	12.4	2.52	2.41
98/99: S	24,433	1044	23	2,491	10.2	2.65	2.48
99/00: F	25,191	1022	25	2,846	11.3	2.57	2.43
99/00: S	24,064	1004	24	2,603	10.82	2.68	2.53
00/01: F	24,797	1051	24	3,094	12.48	2.69	2.57
00/01: S	23,745	1024	23	2,945	12.4	2.75	2.63
01/02: F	24,808	1038	24	3,606	14.54	2.73	2.59
01/02: S	24,134	1035	23	2,426	10.05	2.78	2.59
02/03: F	23,116	973	24	2,311	19	2.72	2.56
02/03: S	21,606	933	23	2,046	9.5	2.79	2.64
03/04: F	22,279	946	24	2,427	10.9	2.71	2.55
03/04: S	20,462	900	23	2,058	10.06	2.73	2.56
04/05: F	20,977	941	22	2,555	12.18	2.67	2.51

04/05: S	18,964	875	22	2,227	11.74	2.77	2.63
05/06: F	19,909	902	22	2,391	12.1	2.73	2.58
05/06: S	17,929	835	21	1,705	9.6	2.79	2.59
06/07: F	20,015	909	22	2,171	10.85	2.75	2.6
06/07: S	18,014	830	22	1,680	9.326	2.79	2.62
07/08: F	21,369	937	23	2,304	10.78	2.74	2.6
07/08: S	20,400	925	22	2,280	11.18	2.77	2.63
08/09: F	22,232	959	23	2,676	12.04	2.74	2.59
08/09: S	19,866	887	22	1,947	9.8	2.79	2.0
09/10: F	23,329	999	23	2,921	12.52	2.74	2.58
09/10: S	21,329	922	23	2,518	11.81	2.8	2.67
10/11: F	22,516	961	23	2,399	10.65	2.69	2.5
10/11: S	20,818	927	22	2,100	10.09	2.82	2.65
11/12: F	22,764	977	23	2,321	10.2	2.83	2.67
11/12: S	21,490	970	22	1,924	8.95	2.78	2.61
12/13: F	23,106	1000	23	2,499	10.82	2.79	2.63
12/13: S	21,583	957	23	1,858	8.61	2.79	2.6
13/14: F	22,716	1003	23	2,104	9.26	2.83	2.61
13/14: S	21,049	973	22	1,712	8.13	2.81	2.62
14/15: F	22,531	981	23	1,987	8.82	2.8	2.57
14/15: S	21,199	934	23	1,639	7.73	2.79	2.58
15/16: F	22,486	967	23	2,221	9.88	2.84	2.66
Total	908,078	39,337	23	98,940	10.89%		

Note. "F" and "S" stand for fall and spring semesters, respectively.

Table 5

Means of GPA, π, and GAI by AFs

AFs	GPA1	GPA2	GPA3	$\pi 1$	$\pi 2$	GAI1	GAI3
Marketing	3.1	3.19	3.27	2.69	5.65	283	281
Physical Education	3.25	3.35	3.45	4.6	8.58	266	265
Spanish	2.7	2.78	2.93	5.27	8.09	283	282
Management	2.87	2.95	3.1	5.42	8.15	285	285
Social Sciences	2.86	2.97	3.2	5.98	8.81	287	286
Education	3.06	3.21	3.45	6.18	10.02	273	273
Finance	2.69	2.89	3.22	6.72	10.32	290	289
Humanities	2.87	2.93	3.11	7.42	10.1	285	286
English	2.81	2.95	3.28	7.56	10.61	285	284
Materials Management	2.5	2.56	2.7	7.8	11.71	276	275
Office Systems	2.62	2.75	2.96	8.09	13.45	259	259
Engineering Technologies	2.31	2.48	2.81	10.66	16	262	265
Physics	2.31	2.56	3.02	10.79	17.23	296	301
Engineering Transfers	2.78	2.92	3.24	10.88	15.68	327	329
Computer Sciences	2.63	2.76	3.07	11.22	15.71	291	293
Biology	2.33	2.45	2.93	11.43	14.25	295	302
Electronics	2.61	2.76	3.06	12.16	18.19	283	280
Accounting	2.52	2.61	3.02	14.83	18.01	291	293
Chemistry	2.17	2.21	2.64	14.93	16.48	301	306
Economics & Statistics	2.22	2.28	2.83	18.31	20.59	291	285
Mathematics	1.71	1.73	2.16	29.22	29.89	291	293

Note. Figures under GPA1, GPA2 and GPA3 correspond to subsample where $W \geq 1$, the full sample, and the subsample where $W = 0$, respectively. $\pi 1$ comes from the full sample while $\pi 2$ comes from the subsample of courses with at least one W. Finally, GAI1 and GAI3 come from the full sample and the subsample without W, respectively.

Table 6 reports several key facts of π by AFs. The service departments responsible for offering the highest number of courses were English (3,922), Mathematics (3,784), Spanish (2,948), and Humanities (2,903). The value in Mathematics was the greatest (29.22%), while the respective figures in the English, Humanities, and Spanish courses were 7.57%, 7.08%, and 5.19%. Two other service programs exhibiting high π values were Economics & Statistics (17.38%) and Chemistry (15.14%). The last column of Table 6 transforms withdrawals (*W*) into equivalent courses by AFs. The exercise requires dividing each value of *W* by the average course size of the respective AFs. The total *W* (98,940) observed during the period would require offering 4,239 equivalent courses to satisfy future demand.

Estimating the costs will be necessary to gauge withdrawals' economic and academic consequences. The approach suggested by Matos-Díaz (2018), assuming that equivalent courses were offered by part-time faculty, paid through the mechanism of additional compensation ($2,000 per course), allows estimating the lower-bound monetary cost of the total withdrawals of around $8.48 million (4,239 x $2,000 = $8,478,000). However, their actual cost might be significantly higher. The 4,239 equivalent courses are more than the total courses offered by service departments such as English (3,922) and Mathematics (3,784) and more than all the courses offered jointly by six different programs.[12] That is, withdrawals entail a waste of resources greater than the whole budget assigned to and spent by such programs during 20.5 consecutive academic years. This is, indeed, a significant waste of scarce resources.

Predicting π, σ^2 (*W*) C_S, and C_S,

Thus far, the discussion has centered on the characteristics of the parameters distributed by AFs and over time. This section is devoted to discussing the results of the estimated models and their policy implications. It was shown that π and σ^2 (*W*) move

[12] The programs are Material Management (309), Engineering Transfers (405), Marketing (738), Finance (791), Economics & Statistics (875) and Chemistry (1,084), for a total of 4,202 courses.

Table 6

Stylized Facts of Withdrawals by AFs

AFs	Enrollment (A)	Courses (B)	Course Size (C = A/B)	*W* (D)	% *W* (D/A)100	Equivalent Courses (D/C)
Marketing	19,696	738	27	523	2.66	19 (0.45%)
Physical Education	40,195	1,970	20	1,859	4.62	93 (2.19%)
Spanish	79,305	2,948	27	4,118	5.19	153 (3.61%)
Management	38,849	1,423	27	2,102	5.41	78 (1.84%)
Social Sciences	68,435	2,551	27	4,085	5.97	151 (3.55%)
Education	60,090	2,714	22	3,850	6.41	175 (4.13%)
Finance	20,112	791	25	1,403	6.98	56 (1.32%)
Humanities	77,191	2,903	27	5,462	7.08	202 (4.77%)
English Materials	95,682	3,922	24	7,242	7.57	302 (7.12%)
Management	7,038	309	23	564	8.01	25 (0.59%)
Office Systems	27,710	1,780	16	2,311	8.34	144 (3.4%)
Engineering Transfers	6,568	405	16	697	10.61	44 (1.04%)
Engineering Technologies	18,418	1,203	15	2,068	11.23	138 (3.26%)
Physics	25,178	1,321	19	2,880	11.44	152 (3.59%)
Computers	45,741	2,496	18	5,305	11.6	295 (6.96%)
Biology	43,332	1,806	24	5,123	11.82	213 (5.02%)
Electronics	38,599	2,471	16	5,187	13.44	324 (7.64%)
Accounting	41,669	1,843	23	5,973	14.33	260 (6.13%)
Chemistry	28,845	1,084	27	4,367	15.14	162 (3.82%)
Economics & Statistics	23,895	875	27	4,153	17.38	154 (3.63%)
Mathematics	101,530	3,784	27	29,668	29.22	1,099 (25.93%)
Total	908,078	39,337		98,940	10.89%	4,239 (100%)

in the same direction until $\pi = 0.5$; then, σ^2 (W) decreases for all $\pi > 0.5$. Likewise, C_S is an entirely decreasing function of π, while C_K decreases until $\pi = 0.5$, and then increases unboundedly. Based on these analytical results, the coefficients of Models 1 and 2 in Table 7 should be expected to share the same pattern of signs. Likewise, the coefficients of Models 3 and 4 should also share the same signs. However, the pattern of signs of Models 1 and 2 should be the opposite of Models 3 and 4, and vice versa, except for values of $\pi > 0.5$ in the two mentioned cases. All semi-continuous covariates, as well as almost all the dummies, satisfy this condition of consistency in the pattern of signs.

The baseline model estimates the equation described in (8) as a first approximation, using the following covariates: summer, SET, GAI, the variance of GAI, the proportion of private high school students, the proportion of female students, and the constant term.[13] The adjusted R-squared for the four models were 0.03, 0.03, 0.05, and 0.04, respectively. When the models were re-estimated accounting for UFH,[14] through fixed-effects models, the coefficients increased to 0.44, 0.41, 0.41, and 0.34, respectively. That is, the total variation around the means explained by the models increased by a factor of 14.67, 13.67, 8.2, and 8.5 times, respectively (results are available upon request). Thus, UFH plays a significant role in the student's decision process related to course withdrawal.

Later, the four models were estimated using all the covariates in Table 7 without accounting for UFH. Almost all AFs' covariates are statistically significant and exhibit the expected pattern of

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13 In order to simplify the discussion of the coefficients in Table 7, hereafter the results belonging to the dependent variables π, σ^2 (W), C_S and C_K will be referred to as Model 1 through Model 4, respectively.

14 UFH was modeled as both random- and fixed-effects. However, according to the Hausman test, the fixed-effects model is preferable to that of random-effects. The assumption of no correlation between the error term (ε_{jf}) and the explanatory variables is rejected at the 0.0000 significant level. Thus, the random-effect estimates are omitted, but they are available on request. It should be mentioned that the fixed-effects model is unable to provide estimates of time-invariant covariates, such as female.

Table 7

Predicting the Parameters of the Distribution of Withdrawals

Variables	π	σ^2 (*W*)	C_S	C_K
Constant	11.562**	8.4462**	2.2733**	7.4762**
	(1.7311)	(1.0384)	(0.2656)	(1.771)
Assistant Professor	0.0673	0.1678	-0.0047	-0.0364
	(0.2584)	(0.1531)	(0.035)	(0.2064)
Associate Professor	0.5005	0.3891†	-0.0248	-0.0608
	(0.3465)	(0.2011)	(0.0457)	(0.2656)
Professor	1.4391**	1.0245**	-0.1145*	-0.4648
	(0.4423)	(0.2506)	(0.0567)	(0.3291)
Doctorate	0.58*	0.2379	-0.0872*	-0.4489
	(0.2746)	(0.1685)	(0.0442)	(0.2795)
Probation	0.3533	0.122	-0.0193	0.0466
	(0.2642)	(0.1637)	(0.0412)	(0.2512)
Tenured	0.039	-0.0913	0.0273	0.2682
	(0.2983)	(0.1812)	(0.0443)	(0.27)
Class Size 1	-2.1446**	-1.7906**	-0.4454**	-2.7868**
	(0.2113)	(0.1304)	(0.0324)	(0.1739)
Class Size 3	-0.1748	-0.1732*	0.2598**	2.2112**
	(0.1265)	(0.0734)	(0.0189)	(0.1199)
Morning	-0.0631	-0.0233	0.0352*	0.1891†
	(0.1168)	(0.0698)	(0.0167)	(0.1004)
Night	-0.8093**	-0.0673	0.0825*	0.1719
	(0.2341)	(0.1371)	(0.0335)	(0.1997)
Summer	-8.8047**	-4.0522**	1.5309**	7.6149**
	(0.6902)	(0.4181)	(0.1984)	(1.1182)
SET	-0.5178**	-0.3282**	0.1082**	0.6304**
	(0.1523)	(0.095)	(0.0239)	(0.148)
Professor's age (Z)	2.3225*	1.4234*	-0.3832**	-2.2538**
	(1.1316)	(0.6031)	(0.1313)	(0.8173)
GAI (Z)	-1.5565**	-0.8608**	0.1702**	0.8515**
	(0.08)	(0.0455)	(0.0105)	(0.0614)
GAI Variance (Z)	-0.184**	-0.0881**	0.0183*	0.0788†
	(0.0526)	(0.0314)	(0.0078)	(0.0477)
Proportion of private school students (Z)	-0.1364*	-0.016	0.023**	0.0849†
	(0.0556)	(0.0342)	(0.008)	(0.048)
Proportion of female students (Z)	-1.5736**	-0.8913**	0.1759**	0.8213**
	(0.0875)	(0.0496)	(0.0115)	(0.067)
Adjusted *R*-square	0.46	0.43	0.43	0.37
Sample size	39,143	39,143	28,046	28,046

Note. †, *, ** Statistically significant at the 10, 5, and 1 percent levels, respectively. Z = standardized variable. Standard errors (in parentheses) are corrected for heteroskedasticity and contemporaneous correlation. Models also control for weekdays (5 dummies), terms (40 dummies), AFs (20 dummies), and UFH through fixed-effects models.

signs previously discussed. Nonetheless, the adjusted *R*-squared coefficients are 0.34, 0.31, 0.32, and 0.26, respectively. These coefficients are even smaller than those reported for the baseline models after accounting for UFH (0.44, 0.41, 0.41, and 0.34, respectively). However, the process of re-estimation of the models accounting for UFH gives rise to the statistical insignificance of a great proportion of the AFs' estimated coefficients. As Table 7 reports, the adjusted *R*-squared coefficients were 0.46, 0.43, 0.43, and 0.36, respectively. This result illustrates the superiority of UFH over AFs covariates.[15] Table 7 reports (in parentheses) the standard errors, corrected for heteroscedasticity and contemporaneous correlation, of all models; however, for space limitations, the table does not report the AFs' coefficients, even though they were included in the four models.[16]

Among the dummies controlling for faculty characteristics, the associate professor exhibits the correct pattern of signs, but it is marginally significant and positive only in Model 2. Professor covariate shares the correct pattern of signs (positive in Models 1 and 2 and negative in Models 3 and 4) in all estimated models. It is significant in Models 1, 2, and 3 but insignificant in Model 4. On the other hand, assistant professor, probation, and tenure are insignificant in all models; while doctorate exhibits the appropriate pattern of signs in all models, it is significant only in Models 1 and 3.

The professor's age covariate could capture the effects of two different scenarios. On the one hand, the course withdrawals may be significant and directly related to the young faculty's lack of teaching skills. If so, they should tend to diminish to the extent that faculty members improve their teaching skills over their academic career life cycle. On the other hand, it might be the case that withdrawals were significant and directly related to intrin-

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15 The null hypothesis stating that the fixed-effects are redundant in all estimated models should be rejected. The estimated cross-section F and Chi-square statistics are highly significant at conventional levels. For technical details, refer to *EViews* (Quantitative Micro Software, 2009).

16 See *EViews* (Quantitative Micro Software, 2009) for details.

sic course difficulty level rather than to a lack of faculty teaching skills. Under such a scenario, it should be expected that during their first years of teaching, new faculty members were subject to pressure from students (through SET) and administrators to grade more leniently. However, such pressure tends to diminish to the extent that faculty get tenure and promotions to higher ranks. If so, course withdrawals and the professor's age will be expected to move in the same direction. Evidence points to the conclusion that the second scenario prevails at UPR-Bayamón because the covariate is significant and exhibits the correct pattern of signs in all models (positive in Models 1 and 2 and negative in Models 3 and 4). Increases of one standard deviation in this covariate will induce increases of 2.32 and 1.42 points in π and σ^2 (W), as well as decreases of 0.38 and 2.25 points in C_S and C_K, respectively.

Almost all the covariates that define the section characteristics, such as course size, hour, and weekdays, as well as summer and SET, are statistically significant. Compared to the reference group (13 to 29 students per course), π and σ^2 (W) decrease by 2.14 and 1.79 points, while C_S and C_K decrease by 0.45 and 2.79 points in smaller courses, respectively. This result is at odds with the expected pattern of signs since the signs of C_S and C_K should be the opposite. One plausible explanation is that the smallest courses have been designed to accommodate students with special academic needs. There are 3,939 courses with enrollment less than or equal to 12 students. Among them, there are 2,256 where $\pi = 0$, and 1,683 where average $\pi = 18.24\%$. If the first set consists of academically lagging students enrolled in remedial courses, while the second set is composed primarily of advanced students placed in small groups of the most difficult or advanced courses, then it will be very unlikely that the models could disentangle the relationship between course size and academic achievement.

On the other hand, in bigger courses (30 or more students), the pattern of signs is consistent (positive in Models 1 and 2 and negative in Models 3 and 4), and the covariate is statistically insignificant only in Model 1. Furthermore, compared to the reference group, $\sigma^2\ (W)$ tends to decrease by 0.17 points, while C_S and C_K tend to increase by 0.26 and 2.21 points, respectively. Therefore, π and course size move in opposite directions. This result has policy implications since the institution would be able to design strategies to identify in advance students with high probabilities to withdraw from determinate courses and try to place them in smaller courses with academic support.

Compared to courses offered in the afternoon C_S and C_K tend to increase by 0.04 and 0.19 points in morning courses, respectively. Meantime, π and $\sigma^2\ (W)$ move in the opposite direction, but their coefficients are statistically insignificant. On the other hand, in the evening courses, π decreases by 0.81 points and C_S increases by 0.08 points; however, the covariate was insignificant in the case of Model 2 $(\sigma^2\ (W))$ and Model 4 (C_K). One possible explanation for such results could be the traffic congestion confronted by students enrolled in courses scheduled early in the morning or the lack of sufficient parking spaces. Both situations could increase late arrivals to classes and absenteeism among students, which in turn would increase π. If such problems have occurred, their frequency seems to be significantly smaller for evening courses. If so, the problem could be mitigated by improving the schedule of the academic offering according to students' needs.

A great proportion (14/20 = 70%) of the weekday dummy covariates is statistically significant. Nonetheless, the pattern of signs of the estimated coefficients is inconsistent. To shed more light on this issue, it would be convenient to increase the specificity level of the analysis considering interactions among hours, weekdays, and level of courses by AFs. Such a task will require further research.

The summer covariate exhibits the expected pattern of signs and is highly significant in all estimated models. After accounting for UFH, π and $\sigma^2\ (W)$ decrease by 8.8 and 4.05 points; meantime, C_S and C_K increase by 1.53 and 7.61 points if the course was offered during the summer. Other things being equal, π diminishes dramatically in summer sessions. This result becomes more pertinent when considering that Mathematics courses (346) represented 79% of all summer courses (440). According to student performance, Mathematics courses are the most difficult. Their overall GPA and π in fall and spring terms are 1.71 and 29.22%, respectively. However, the respective figures for summer sessions are 2.10 and 14%. Thus, GPA increases by 23% and π decreases by 52% if the course is taught in summer. These numbers explain succinctly why Mathematics summer courses are so popular among students and why the Mathematics Department's market share of the summer offer is as high as 79%. Given that course inherent difficulty remains equal, no matter the session, there are only two possible explanations: (a) students take fewer courses in the summer and, therefore, can concentrate on a course more intensively, and (b) faculty members grade more leniently, relaxing academic standards possibly to prevent competition for teaching assignments. Both explanations have important policy implications that would require further research.

To empirically test the leniency hypothesis, attention is placed on the SET estimated coefficients. According to this conjecture, faculty members will get better SET ratings if they reduce academic standards and course difficulty levels through leniency grading. Such a symbiotic relationship between students and faculty has been proposed in the literature for a long time without direct statistical testing.[17] If so, it should be expected that in courses where SET = 1, the difficulty level diminishes, the GPA increases, and π decreases.

The SET estimated coefficients are statistically significant and exhibit the expected pattern of signs in all models. For instance, π and σ^2 (W) decrease by 0.52 and 0.33 points; meantime, C_S and C_K increase by 0.11 and 0.63 points if SET were conducted in the course. Other things being equal, π significantly diminishes whenever SET = 1. Therefore, according to students' criteria, inherent difficulty significantly decreases just for the simple reason

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17 For some relevant studies in the field, refer to Dilts (1980), Isely and Singh (2005), Krautmann and Sander (1999), McPherson (2006), Nelson and Lynch (1984), Seiver (1983), and Zangenehzadeh (1988).

that the course is under SET. This result is consistent with the symbiotic relationship conjectured in the leniency hypothesis.

Among the available student quality proxies, GAI is the most relevant because it constitutes the institution's admission policy criterion. Therefore, it should be expected that both GAI and GAI variance exert a significant effect on the four dependent variables under study. Table 7 reports the estimated coefficients showing that such is the case. For example, the estimated coefficients of the GAI covariate are highly significant (with the correct pattern of signs) in the four estimated models. Other things being equal, increases of one standard deviation in student quality (GAI) will induce decreases of 1.56 and 0.86 points in π and $\sigma^2(W)$, respectively. However, C_S and C_K are expected to increase by 0.17 and 0.85 points, respectively. On the other hand, increases of one standard deviation on the GAI variance covariate will induce reductions of 0.18 and 0.09 points in π and $\sigma^2(W)$, respectively. Contrariwise, C_S and C_K are expected to increase by 0.02 and 0.08 points, respectively.

The observed inverse relationship between GAI and π is what should be expected under normal academic circumstances. Notwithstanding, the pattern of signs exhibited by the estimated coefficients of GAI variance covariate needs some further explanations. The heterogeneity of student quality, proxied by this covariate, might have different effects on the dependent variables under study depending on the professor's attitude toward risk. For instance, faced with courses of highly heterogeneous students, a risk-averse professor would relax the academic standards to allow students belonging to the lower bound of the quality distribution to exceed threshold GPA values that induce them to not withdraw from the course. Thus, relaxing academic standards would improve the distribution of grades, reduce π, which, in turn, would increase the probability of better SET ratings for the professor teaching the course. Under such scenario, π and GAI variance should move in opposite directions. Evidence points to the conclusion that this is the case prevailing at UPR-Bayamón. Thus, both variables behave as expected. However, their policy implications are difficult to achieve. For example, other things being equal, to induce a reduction of 3.12 points in π observed in Mathematics courses, it would be necessary to admit new entrance students with GAI two standard deviations above the mean. That is, it would require recruiting students with a GAI of about 333 points. Usually, students with such credentials apply and obtain admission to programs more competitively offered by other campuses of the UPR system or by U.S. universities. In the case of GAI variance covariate, it will be difficult, if not impossible, to control it. Hence, given the institutional official admission policy (GAI), the increases in student quality required to partially offset the observed π by AFs are unfeasible. However, other things being equal, π is expected to diminish by 8.68 points just for the simple reason that the course will be during the summer session. Therefore, the structure of incentive mechanisms prevailing among faculty members and students during summer sessions deserves further research.

Two other student characteristics that could contribute to explaining the variance around the dependent variables of the models are the private high school and female student proportions. Both proportions significantly vary among programs. For the full sample, they are equal to 47% and 53%, respectively. However, for Office Systems, the figures are 32% and 94%, respectively. On the other hand, the respective female proportion in programs such as Education and Biology are 85% and 71%, but in Electronics, it is only 7%. Therefore, students are not randomly distributed among programs.

The female proportion covariate is highly significant in all the models. Other things being equal, increases of one standard deviation on it will be associated with reductions of 1.57 and 0.89 points in π and σ^2 (W), respectively. Meanwhile, it is expected that C_S and C_K increase by 0.18 and 0.82 points, respectively. On the other hand, increases of the same magnitude in the proportion of private high school students will induce a decrease of 0.14 points in π, as well as increases of 0.02 and 0.08 points in C_S and C_K, respectively. The coefficient is statistically insignificant in the case of Model 2 (σ^2 (W)). Thus, to the extent that both proportions tend to increase, π decreases significantly. Given that the control of both variables is beyond institutional reach, there is no space to use them as a policy mechanism design.

The inclusion of a set of forty time-varying dummies, which uses the first term as the reference group, allows us to capture the effect of time on the dependent variables of the models. The purpose was to evaluate whether the estimated models might mimic the growth path exhibited by the key parameters depicted in Figure 2. Although Table 7 does not report the estimated coefficients, a significant proportion is statistically significant and exhibits the expected pattern of signs in all models. For instance, nineteen out of 40 (48%) of the estimated coefficients of Model 1 and 25 out of 40 (62.5%) of Model 2 were significant, and their pattern of signs is the expected one (negative), according to Figure 2. On the other hand, the respective proportion for Models 3 and 4 is 65% for each one (26/40), and the pattern of signs is the expected one (positive), according to Figure 1. Thus, the time-varying coefficients of the four estimated models mimicked the exhibited growth path of the dependent variables very well.

Summary and Conclusions

Using a rich panel containing detailed information on the 39,337 courses offered during forty-one consecutive terms, this study analyzed the distribution of course withdrawals and its key moments at the UPR-Bayamón. Overall, the fit of the estimated models is very good. Evidence shows that courses, faculty, and students' characteristics exert a strong and significant influence on π, σ^2 (W), C_S and C_K. UFH, captured through random- and fixed-effects models, explains a significant proportion of the variation observed around the dependent variable of each estimated model. Empirical evidence does not allow rejection of the symbiotic relationship between faculty members and students, conjectured in the literature, since the estimated coefficients of the SET covariate were highly significant and exhibited the correct pattern of signs in all models. That is, π and σ^2 (W) tend to decrease, while C_S and C_K tend to increase for the simple reason that the SET was conducted in the course.

A similar result was observed in the case of summer covariate. Its estimated coefficients were highly significant in all estimated models. As discussed previously, π is expected to diminish by 8.8 points if the course is offered during the summer session. However, under the unlikely scenario that the institution would be able to recruit new entrant students with a GAI two standard deviations above the mean (GAI about 333 points), π would decrease by only 3.1 points. Hence, offering a Mathematics course during the summer session would have an expected effect on π equivalent to admitting new entrant students with GAI 5.64 standard deviations above the mean, which is impossible. Therefore, the signs and significance of the coefficients of GAI, SET, summer covariates, and UFH have important implications for the institution's academic policy mechanism design. Empirical evidence points to the conclusion that at UPR-Bayamón, there exists an environment where faculty members and students engage in a *shopping-around* process where both parties improve their well-being at the expense of reductions in academic standards and the quality of the education provided. Under such a scenario, it might be possible to explain the contradictions observed in the institution where, even though the indicators of student quality are consistently decreasing over time, the GPAs are increasing and π is decreasing simultaneously.

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