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A modeling approach of return and volatility of structured investment products with caps and floors

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ABSTRACT
A popular investment structured product in Puerto Rico is the stock market-tied Individual Retirement Account (IRA), which offers some stock market growth while protecting the principal. The performance of these retirement strategies has yet to be studied. This work examines the expected return and risk of Puerto Rico stock market IRAs (PRIRAs) and compares their statistical properties with other investment instruments after tax. We propose a parametric modeling approach for structured products and apply it to PRIRAs. Our method first estimates the conditional expected return (and variance) of PRIRA assets from which we extract marginal moments through the Law of Iterated Expectation. Our results indicate that PRIRAs underperform against investing directly in the stock market while carrying substantial risk. PRIRAs may be reasonable for some risk-averse investors due to their principal protection and tax deferral.

Keywords: individual retirement accounts, Puerto Rico Investment Companies Act, passive investing, stock market, bond market

Un modelo del rendimiento y la volatilidad de productos de inversión estructurados con límites máximos y mínimos

RESUMEN
Un producto estructurado de inversión popular en Puerto Rico es la Cuenta de Retiro Individual (IRA) vinculada a la bolsa de valores. Estas ofrecen cierto crecimiento atado a la bolsa de valores mientras mientras protegen el capital. Este trabajo examina, por primera vez, el rendimiento esperado y el riesgo de las IRAs de la bolsa de valores de Puerto Rico (PRIRAs) y compara sus propie-

dades estadísticas con otros instrumentos de inversión después de impuestos. Proponemos un modelo paramétrico y lo aplicamos a PRIRA. Nuestros resultados indican que las PRIRAs tienen un rendimiento inferior al de invertir directamente en la bolsa de valores, aunque conllevan un riesgo relativamente alto dado su rendimiento. Las PRIRAs pueden ser razonables para algunos inversores reacios al riesgo debido a su protección del principal y aplazamiento de impuestos.

Palabras clave: cuenta de retiro individual, Acta de compañías de inversión en Puerto Rico, inversión pasiva, bolsa de valores, mercado de bonos

Introduction

A structured investment product, also named market-linked investment, is one of the most popular financial derivatives. The payoff at maturity relies on one or more specified assets (mostly stocks or stock indices). Because structured products emphasize downside protection with simultaneous participation in the upside, they are very attractive to retail investors (Blundell-Wignall, 2007). However, banks may design complex products for yield-seeking households (Bordalo et al., 2016). Based on the Principal-Protected Note (PPN) principle, some Puerto Rican institutions create prepackaged individual retirement accounts with returns tied to the stock market.

The Individual Retirement Account (IRA) is an investment wrapper solely funded by an individual to save for retirement while reaping some tax benefits. The IRA originated in Chile became standard across Latin America, and is also available in the United States. European countries have similar products such as Lifetime Individual Savings account (U.K.), Rester Rente (Germany), and more recently, pan-European personal pension products for the European Union. Unlike in the U.S., the IRAs in Puerto Rico are prepackaged, with their returns being pegged to some other asset. Most financial institutions in Puerto Rico have IRAs pegged to a certificate of deposit or offer some low return. These versions of IRA have fixed known returns. We aim to focus on Puerto Rico stock market IRAs(PRIRAs), which are pegged

to the performance of the S&P 500. Retirement products such as PRIRA are not common in places other than Puerto Rico, and indeed, no evidence of them elsewhere was found.

Moreover, more information is needed about the statistical properties of these assets. Puerto Rico has been going through an economic recession since 2006. Leaders on the island have been attempting to find ways to boost the economy (Rivera, 2016), and data-driven methods may help improve investment opportunities (Lugo & Rivera, 2023; Rivera, 2020; Rivera et al., 2019; Rivera et al., 2020; Rivera & Rolke, 2019; Rivera & Rosenbaum, 2020; Rosenbaum et al., 2021). Since 2013, the government has been forced to reform its deficient public employee pensions. Under the pension reforms, there was a noticeable increase in the retirement age and employee contributions (Austin, 2016). However, most Puerto Ricans seem unprepared to take charge of financial decisions and retirement (Castro-González, 2014).

Although research on different aspects of structured products exists, prior studies investigating the structured products tied to Puerto Rican IRA and their statistical properties have yet to be found. As a structured product, PRIRAs not only provide principal protection but also come with the usual tax benefits of individual retirement accounts. The Puerto Rico Investment Companies Act, which was approved in 1994, states that 67% of the capital of an investment company should be invested in local securities. Although this law is intended to promote economic growth, the limited options for local companies to invest greatly hinder the local investment companies' options to construct their investment instruments. This makes it difficult for investment companies to construct investment alternatives. Local investment products, like PRIRAs, may have specific uncommon properties outside Puerto Rico. The importance of this study is to examine the expected return and risk traits of the PRIRAs before- and after-tax and make some comparisons with other investment instruments. This study's findings will help guide many future decisions made by investors and pundits.

Puerto Rico IRA systems, both traditional and Roth plans, dif-

fer significantly from U.S. IRAs. Income from non-federal employment within Puerto Rico cannot be contributed to a U.S. IRA. Notably, the U.S. IRA is usually presented as an empty box. Most investors can place anything (stocks, bonds, index funds) up to $6,000 (currently) in the IRA. As a result, they are exposed to investment risks. In contrast, in Puerto Rico, IRAs are a prepackaged investment instrument. It either comes with fixed returns, for example, using C.D.s, or may offer returns contingent on something like the S&P 500. This study examines two IRA products, one from Popular Bank (Banco Popular) and the other from Universal Insurance Company (Universal). Both are the typical structured products whose returns are determined by the performance of the S&P 500 index. Issuers protect the invested principal.

The PRIRAs are Structured Certificates of Deposit (SCDs), whose payoffs depend on the performance of the S&P 500 index. If the underlying asset does well, the payoff from the SCD will be limited to a predetermined cap, which a particular SCD specifies. However, the SCDs can guarantee that the principal will be returned at the maturity date even though the underlying financial assets perform poorly. In particular, the SCDs are typically worth about 93% of the value of a contemporaneously issued fix-rate CD (Deng et al., 2013). Generally, the SCDs are constructed by a risk-free asset combined with Asian call options to obtain the desired payoff. $PRIRA_1$ is made up of three components. The first one, a zero-coupon CD issued at a discount to par value with a five-year maturity. Popular Bank can get funding from issuing this product. The cost of funds will depend on the discount and its implied accretion to par. The second component is a long call Asian option at the money of the S&p 500 (strike price = 100% of the initial index value). This option will be a cost to the bank. The last component is a short call Asian option out of the money on the S&P 500 (strike price = 125% of the initial index value). The option will provide a credit to the bank. Moreover, this option effectively caps the upside the client will receive because he/she purchases the 100% call option and sells the 125% call option.

The information about how Universal constructs the $PRIRA_2$ is considered privileged and has not been shared with the public, yet we believe it works similarly to $PRIRA_1$.

Methodology

This section briefly describes our statistical framework and data for assessing the PRIRAs' performance. For more details, refer to He and Rivera (2023).

Volatility Model

We model each $PRIRA_i$'s (i=1, 2) expected return and risk, applying the truncated normal distribution and the iterated expectation model. The latter incorporates the variance of the returns in the estimation. We will compare $PRIRA_i$'s statistical properties with those of other investment instruments, such as deposit savings, an index fund that follows the S&P 500, a U.S. bond index fund, and a diversified portfolio comprising U.S. stocks and bonds. We account for tax effects on the expected return and risk.

Comparison Benchmarks

We aim to compare PRIRAs' performance with other approaches to determine the best long term investment. Specifically, we chose four other common investment instruments to compare: deposit savings, an index fund that follows the S&P 500, a U.S. bond market index fund, and a diversified portfolio of stocks and bonds.

We will use proxies of these investment strategies. Current deposit account interest rates in Puerto Rico will be obtained to model returns from deposit savings, Vanguard 500 Index Fund Investor Shares (VFINX) will represent index fund, Vanguard Total Bond Market Index Fund Investor Shares (VBMFX) will represent bond market fund, and the diversified portfolio (of Vanguard stock and bond index funds) will represent a diversified portfolio. We assume the annual expected return of a savings

account is 0.25%. We will use the sample mean and standard deviation for the annual expected return and risk for index funds, bond market funds, and diversified portfolios.

We assume investment horizons of 10 and 30 years. To ease the return comparison for each strategy, we arbitrarily assume that the investor invests $10,000 per year. Note that for the IRA, this implies that the investment strategy is for a couple who files jointly. We use the 10-year U.S. T-bond yield as the risk-free rate to calculate the Sharpe ratio (Sharpe, 1966). The average 10-year U.S. T-bond over all periods is about 5.96% (Damodaran, 2024). We account for tax adjustments in our comparison (for details refer to He & Rivera, 2023).

Data

This study used the closing prices of the S&P 500 to measure stock market value. Moreover, we recorded the annual return rate and dividend return rate of the stock market index fund and the bond market index fund. All the data was downloaded from the Center for Research in Security Prices.

Daily closing prices of the S&P 500 were used to establish the initial index value and monthly end index value from April 1957 to March 2019 to calculate $PRIRA_1$ returns. When analyzing $PRIRA_1$, we arbitrarily assumed that the opening date was April 2nd of each year. The closing prices of the S&P 500 every April 1st from 1957 to 2014 were collected and noted as each initial index value. We also collected the closing price of the S&P 500 on the last business day of each month during the term as the monthly end index value. For $PRIRA_2$, we collected the value of the S&P 500 index for every anniversary. We assumed that the policy's first anniversary was April 1st, 1957. The time range for $PRIRA_2$ was from 1957 to 2019.

For VFINX and VBMFX, we computed the annual total return rate to calculate the expected annual return and the risk of return from 1987 to 2019. When considering the tax effect on VFINX and VBMFX, we needed to focus on the annual dividend return. VFINX and VBMFX were used as proxies to measure the performance of a diversified portfolio.

Results

In this section, we present the implementation of our statistical framework to compare the performance of the PRIRAs with other investment alternatives. The computational aspects of our volatility model are detailed in He and Rivera (2023). Table 1 presents the estimated expected return, standard deviation, and Sharpe ratio after-tax for each investment at different tax rates.

Table 1

After-tax comparisons for different investment instruments

	Tax Rate	E(R)	σ(R)	Sharpe ratio
$PRIRA_1$	/	4.7%	8.5%	-14.8%
$PRIRA_2$	/	4.45%	4.9%	-30.82%
Savings Account	0%	0.25%	/	/
	10%	0.225%	/	/
Stock Index	0%	11.9%	17%	34.9%
	7%	11.7%	17%	33.7%
	14%	11.6%	17%	33.2%
	25%	11.3%	17%	31.4%
	33%	11.1%	17%	30.2%
Bond Index	0%	6%	4.7%	0.85%
	7%	5.6%	4.7%	-7.6%
	14%	5.2%	4.6%	-16.5%
	25%	4.6%	4.5%	-30.2%
	33%	4.2%	4.5%	-39.1%
Diversified Portfolio	0%	10%	12.2%	33%
	7%	9.8%	12.2%	31.4%
	14%	9.6%	12.1%	30%
	25%	9.2%	12.1%	26.7%
	33%	9%	12.1%	25%

Note. VFINX represented the stock index portfolio, while VBFMX represented the U.S. bond index portfolio.

In the second column of Table 1, the 0% tax rate corresponds to the before-tax values of the statistics. Stock and Bond index returns before tax are slightly higher than the historical returns of stock and bond funds (McCarthy & Tower, 2021). As we can see, the Stock index portfolio had the largest positive after-tax expected return (about 11%), and the savings account had the lowest (0.225%). The annual expected return of the diversified portfolio after tax was about 9%, which was approximately 2% lower than that of the Stock index, yet three times higher than the average inflation. Moreover, the annual expected return of the Bond index after tax was about 5%. The annual expected returns of PRIRAs after tax were clearly lower than that of the Stock index and diversified portfolios. The expected returns of the Bond index after-tax, at 7% and 14%, were greater than that of $PRIRA_1$. Of note es that, the annual expected return of the Bond index after-tax, at 25% and 33%, underperformed $PRIRA_1$, while only the expected return of the Bond index after-tax, taxed at 33%, underperformed $PRIRA_2$.

The fourth column presents a small tax effect on the risk volatility of the Bond index and the diversified portfolio. Specifically, the standard deviations of the Bond index and the diversified portfolio decreased when the tax rates increased. On the other hand, the tax had no effect on the standard deviation of the Stock index portfolio.

In the fifth column, the Sharpe ratio of the Stock index, the Bond index, and the diversified portfolio decreased with the increased tax rates. It indicated that with the increased tax rate, the Stock index, Bond index, and diversified portfolio achieved less return per unit of risk. Despite the reduction in expected returns in the Stock index and diversified portfolios, the Sharpe ratio of the Stock index and the diversified portfolios were positive and far greater than that of PRIRAs. Note that the Sharpe ratios of the Bond index portfolio became negative after being subject to the tax. Furthermore, the Sharpe ratios of the Bond index, which were subjected to 14%, 25%, and 33% ordinary income tax, were less than that of $PRIRA_1$. However, a negative Sharpe

ratio is difficult to evaluate because a negative excess return with a large standard deviation will make the Sharpe ratio less negative. For example, the negative excess return of $PRIRA_1$ (4.7%-5.96%=-1.26%) was similar to that of the Bond index if the Bond index were subjected to a 14% tax (5.2%-5.96%=-0.76%), but the standard deviation of $PRIRA_1$ was about two times than that of the Bond index. Thus, the Sharpe ratio of $PRIRA_1$ was greater than that of the Bond index (the Sharpe ratio of $PRIRA_1$ was less negative), which indicates that $PRIRA_1$ performed better than the Bond index portfolio.

Table 2 presents the expected portfolio values of all investment options based on different ordinary income tax rates after 10 years and 30 years. The interest income tax rate is 10%.

Based on different tax rates, the expected portfolio values of the Stock index portfolio after-tax were highest among six investments after 10 years and 30 years. The expected portfolio values of the diversified portfolio were comparably smaller than that of the Stock index at the same tax rates, yet markedly higher than the PRIRAs. The expected portfolio values of the Bond index portfolio were slightly higher than that of $PRIRA_1$ at 7% and 14% ordinary income tax rates. However, $PRIRA_1$ outperformed the Bond index at 25% and 33% tax rates. The expected portfolio values of the Bond index portfolio were slightly higher than that of $PRIRA_2$ at 7%, 14%, and 25% ordinary income tax rates. Nevertheless, $PRIRA_2$ outperformed the Bond index at a 33% tax rate.

Figures 1 and 2 compare the expected portfolio value based on different tax rates over 10 and 30 years. Note that the expected portfolio value of PRIRAs were slightly below that of the Bond index (purple line) at 7%, and 14% tax rate. Moreover, the expected portfolio value curves of PRIRAs and the Bond index were almost overlapping at the 25% income tax rate. Still, the expected portfolio value of PRIRAs gradually surpassed that of the Bond index at the 33% tax rate.

Table 2

Expected Portfolio Value of Each Investment after Tax after 10 Years and 30 Years Based on Different Tax Rate

Time Period (yrs)	Interest Income Tax Rate (%)	Ordinary Income Tax Rate (%)	$PRIRA_1$	$PRIRA_2$	Savings Account	Stock Index	Bond Index	Diversified Portfolio
10	10	7	129,861.11	118,077.96	101,245.89	192,095.43	133,561.21	171,358.81
		14	129,861.11	118,077.96	101,245.89	192,095.43	133,561.21	171,358.81
		25	129,861.11	118,077.96	101,245.89	188,826.08	129,134.28	167,498.79
		33	129,861.11	118,077.96	101,245.89	186,678.56	126,270.09	165,602.93
30		7	660,820.95	490,026.78	310,693.69	2,543,670.37	778,355.01	1,739,129.88
		14	660,820.95	490,026.78	310,693.69	2,492,793.06	723,422.85	1,671,728.43
		25	660,820.95	490,026.78	310,693.69	2,346,463.82	649,053.13	1,545,153.55
		33	660,820.95	490,026.78	310,693.69	2,253,944.55	604,317.63	1,485,752.17

Figure 1

The Expected Portfolio Value of Each Investment over 10 Years Based on Different Tax Rates

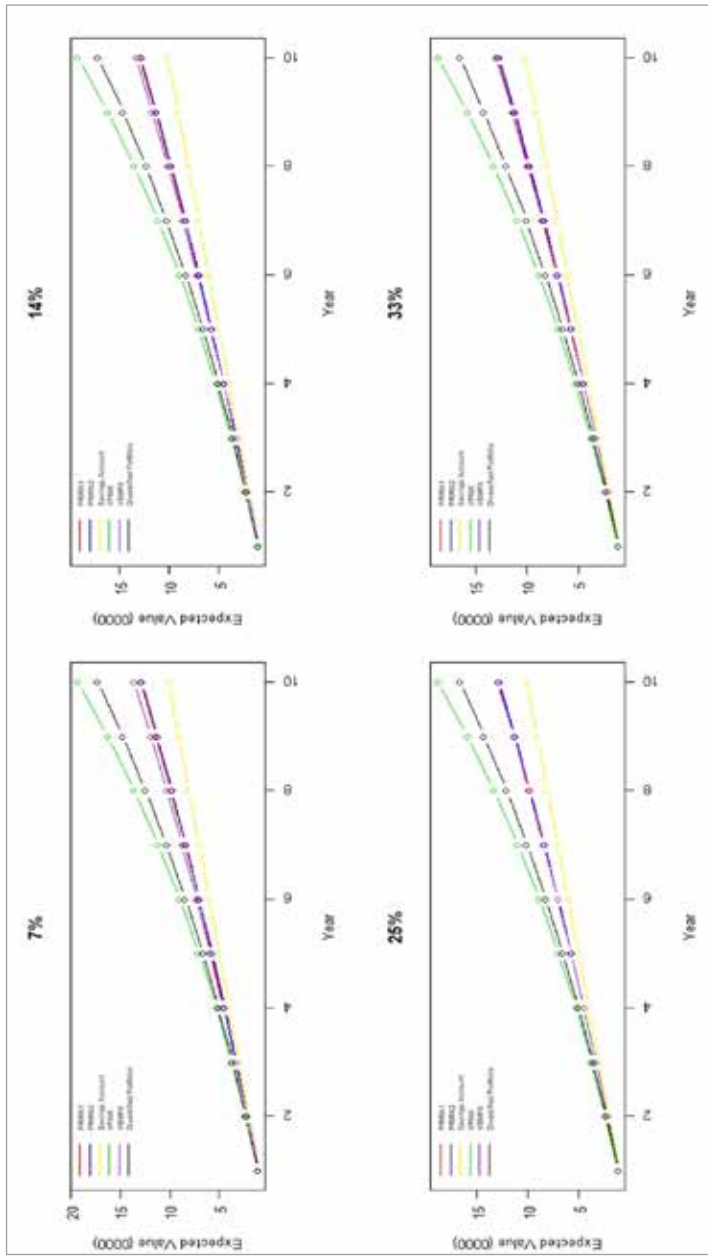
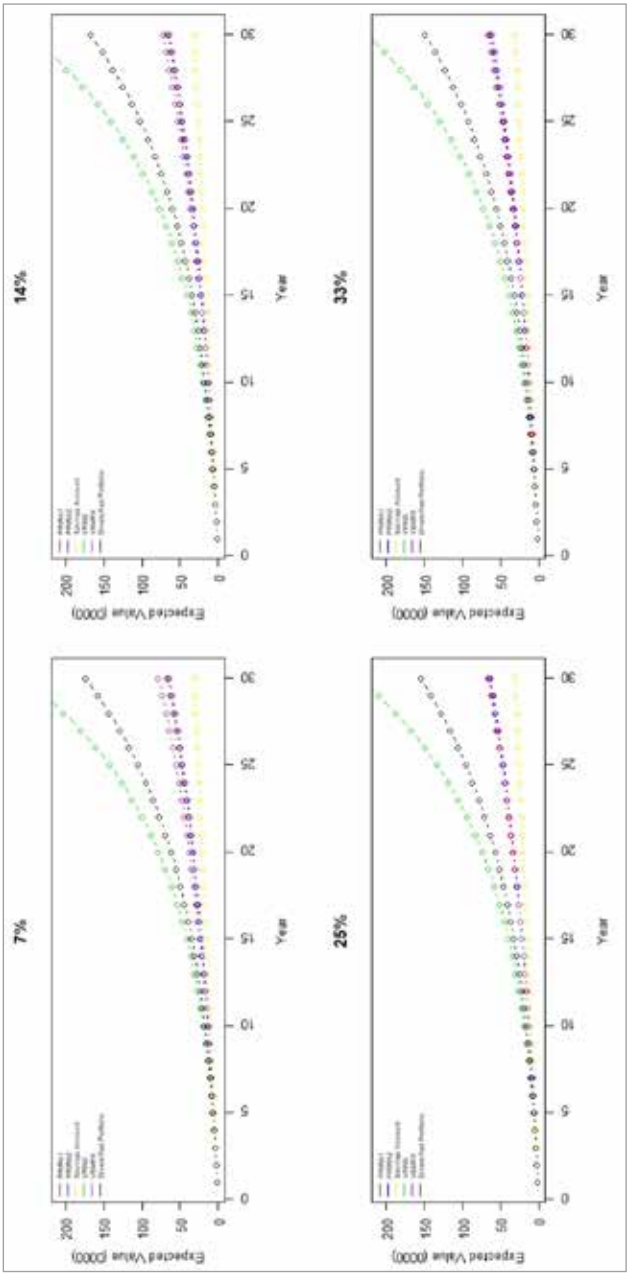


Figure 2

The Expected Portfolio Value of Each Investment over 30 Years Based on Different Tax Rates



Discussion

Our work proposes a statistical framework to model the return and volatility of Puerto Rico IRAs. We then use this framework to estimate the expected return and risk of the PRIRAs and compare their statistical properties to other common investing strategies. The comparisons between PRIRAs and the benchmarks accounted for the tax impact on the expected return and volatility. We focused on passive investing strategies since there is little support of active strategies being worthwhile to most investors (Malkiel, 2016; Sushko, 2018).

The volatility model estimated the annual expected return and standard deviation for $PRIRA_1$ ($E(R)=4.7\%$; $\sigma(R)=8.5\%$) and $PRIRA_2$ ($E(R)=4.45\%$; $\sigma(R)=4.9\%$). Investing in the stock market or a diversified portfolio with stocks and bonds has a higher expected return than PRIRAs. While accompanied by higher risks than PRIRAs, the latter alternatives showed expected returns lower than the risk-free rate. However, $PRIRA_1$ outperformed investing in bonds after tax at 25% and 33% ordinary income tax rates. Although the expected return of $PRIRA_1$ was slightly greater than that of investing in bonds, which was taxed at high tax rates (25% and 33%), the volatility of $PRIRA_1$ return was almost two times that of investing in bonds. The expected return of $PRIRA_2$ outperformed investing in bonds after tax at a 33% ordinary income tax rate, while $PRIRA_2$ had a greater risk. The expected return of $PRIRA_2$ was slightly lower than that of $PRIRA_1$, with less investment risk.

Our findings suggest that, at best, PRIRAs may be reasonable for some risk-averse investors thanks to the principal protection and tax deferral. After considering tax effects, $PRIRA_1$ may provide a higher expected return than the bonds index funds to investors in high tax brackets, albeit suffering about two times more volatility than bonds index funds. Generally, $PRIRA_2$ may be a good investment option for investing purposes with moderate expected return and reasonable investing risk. The increase in risk to investors has resulted in strong demand for insurance products

and supplemental financial products that deliver a guaranteed return on pension savings in many developed countries (Afik et al., 2023; Célérier & Valle, 2017). Also, structured investments, such as the PRIRAs, may be reasonable investment alternatives in countries where investors have limited access to the investment market. However, our results indicate that, for most investors in Puerto Rico, the IRA tax benefits will not lead to a higher expected portfolio value over the long term. While riskier, a diversified portfolio, on average, leads to a portfolio value more than two times higher than PRIRA portfolios over 30 years. Our findings can be attributed to the Puerto Rico Investment Companies Act and how it limits investors' options.

There are some limitations to our research. Considering the tax effects process, we assumed the investor did not withdraw the money from the account. That said, the work clearly illustrates the statistical properties of PRIRAs, assuming the investor invests in the traditional IRA. Nevertheless, it also raises the question of the statistical properties of PRIRAs based on the Roth IRA. Furthermore, we considered the tax effects on different investments, assuming the investor was taxed at Puerto Rican local taxes. Although most Puerto Ricans do not have to pay the federal income tax, some people in Puerto Rico file (jointly) federal taxes, not state taxes. These people can have federal IRAs, therefore not having to pay local taxes. In such cases, investing in stocks, bonds, or a diversified portfolio would be much more beneficial than PRIRAs; the latter would make little sense as an investment vehicle.

Future research includes analyzing the statistical properties of the federal IRA and comparing the differences between the federal IRA and the Puerto Rico IRA. Local institutions may modify the current retirement investment products to satisfy local investors' needs. They may also consider creating more diversified investment vehicles for local investors for retirement savings and investing purposes. Local government should improve residents' awareness of saving money for retirement, which can significantly decrease the future burden of the local retirement system. The Puerto Rican government should reconsider modifying the cur-

rent investment law to facilitate a diversified economy, and incentivize local investment companies to construct more appealing investment products.

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