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MATHEMATICS AND THE CRISIS OF SCIENCE

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Husserl's last book, *The Crisis of European Science and Transcendental Phenomenology*, was written during one of the bleakest periods of European history.[1] European culture, he rightly saw, was in crisis. But more than political, Husserl saw a cultural crisis, which he thought phenomenology could help to overcome, bringing cultural renewal. But, interestingly, Husserl chose to focus his criticism on physical science rather than other aspects of culture. This choice may seem curious, but Husserl believed that the modern science of Nature exemplified paradigmatically what he took to be wrong with modern culture in general. For the crisis of culture was, Husserl thought, essentially a crisis of *meaning*, or lack thereof. So, he saw the supposed manifestation of this crisis in modern physical science (by which we usually mean science from the scientific revolution of the XVII century on) – namely, the fact that a gap lays open between the practices of the modern science of Nature and the sources from where he believed these practices derive their meaning – as a vantage point from where to conduct his diagnosis and suggest the prophylaxis he considered adequate for this supposed cultural malaise of modernity (we can presuppose also, perhaps, that the prominent position physical science has in modern culture and the methodological paradigm it poses for other sciences had a role in his choice of target).[2]

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[1] According to Gérard Granel (introduction to the French translation of *Crisis*), the main text dates from 1935-36 and comes from different sources. Complementary texts have different dates and origins; the first, for instance, ("Science of Reality and Idealization: The Mathematization of Nature"), from 1926-28; and the third ("The Crisis of European Humanity and Philosophy"), which contains the conference of Vienna, from 1935.

[2] Given the success of modern physics, Husserl had to clearly separate its *technical* accomplishments, which he thought must be preserved, from what he took for an "alienated" interpretation of its methods, which he thought must be avoided for the sake of cultural renewal. But, as I hope to establish here, Husserl's remedy for this 'alienation', namely, driving physical science

But how, according to Husserl, this crisis manifests itself in science? Was it related to the exciting new developments that occurred in physics and mathematics not long before the book was written?

In the beginning of the 20th century mathematics was going, and had been going for some time already, through a period of crescent formalization,[3] leaving behind the supposedly solid grounds of intuition[4] and giving free rein to pure formal imagination.[5] Riemann,[6] for instance, had created the theory of general abstract geometrical manifolds; Cantor, the theory of transfinite numbers; Hamilton, the theory of quaternions, and non-commutative algebras in general; Lie, the theory of transformation groups; Grassmann, the theory of extensions.[7] But none of these creations seemed problematic to Husserl; all the above theories were duly appreciated by him, and are mentioned here only because Husserl explicitly praised them (see §§ 69-70 of the *Prolegomena*). But, let us make this clear, not as *scientific* theories in the strictest sense, since these theories do not provide us with *knowledge* of *particular* types of objects, but as formal ontological theories, being as they are theories of purely formal manifolds, i.e. logical *forms* that could in principle, but maybe not actually (or maybe *not yet*) give form to materially determinate domains. Formal mathematical theories are, according to Husserl, theories of objectual domains determinate as to form, but indeterminate as to content (which he named *formal domains*) belonging to a corner of formal logic he called formal ontology.[8] The emergence of formal mathematics (which dates back to the dawn of modern mathematics in the 16th century, with the introduction of the so-called imaginary numbers by the Italian algebraists – if not before, with the creation and development of algebraic thought in the Islamic empire), was definitely not, for Husserl, *in itself*, the cause of any crisis in mathematics.[9]

Physics, on its turn, had by then seen two groundbreaking developments: the theories of relativity (special, 1905; general, 1916), which Husserl did not mention, and quantum mechanics, which he did. Quantum theory is particularly interesting in its use of mathematics, compared with the traditional mathematical sciences of Nature (and this is something to which we should pay attention, since, as already noticed, the mathematization of science is one of the ways by which Husserl saw crisis coming in). Although Born, Jordan, Heisenberg and Dirac had succeeded in formulating quantum mechanics in terms of matrix calculus, this did not follow naturally from a mathematical idealization of the phenomena, a mathematical model of reality that required the matrix approach as the *correct* approach. The creators of quantum mechanics relied simply on *formal* analogies between empirical rules of calculation and properties of matrix operations.[10]

Other mathematical formulations of quantum mechanics, such as wave mechanics, offered even more striking examples of the new use of mathematics. For example, De Broglie's model of the free electron as a particle accompanied by a

back to the "sources from where meaning derives", rooting it firmly in the *Lebenswelt*, can seriously jeopardize its success.

[3] By "formalization" I do not mean axiomatization in the context of formal-logical systems, but the tendency to privilege purely formal mathematical theories; theories that characterize their (purely formal) domains implicitly and independently of any intuitions.

[4] The objectual and conceptual varieties of mathematical intuition are supposed to give us objects and concepts prior to their theories, whose basic truths only express the intuitively given.

[5] Which allows us to invent theories independently of any prior intuition; but maybe answering to other demands, such as, for example, the needs of mathematics and science, the desire to generalize already existing theories – Riemann and Cantor being classical cases –, or the pursuit of "aesthetic" goals such as intrinsic elegance or beauty, etc.

[6] See, for instance, "On the Hypotheses that Lie at the Foundations of Geometry" of 1854.

[7] *Ausdehnungslehre*, basically a forerunner of vector analysis.

[8] According to Husserl a *formal domain* (or manifold) is essentially a structured system of materially indeterminate objects (also called *formal objects*) that function in the system as, basically, placeholders. The structuring relations of formal manifolds are characterized only formally, i.e. independently of the nature of any particular materially determined objects that may take the place of formal objects upon interpretation (which can be construed as the filling of formal objects with material content). We can also define a formal domain simply as the abstract aspect common to all models of a formal theory. For Husserl, a formal domain is the objective correlate of a formal theory.

Husserl's ideas concerning the fundamental distinction between material mathematical sciences, such as geometry and mechanics, and formal mathematical sciences, such as the theory of quaternions, were strongly influenced by Grassmann's views as put forward in *Die lineale Ausdehnungslehre*, 1844. In this work Grassmann makes this distinction clearly, claiming that formal mathematical theories, unlike material ones, are not theories of domains existing independently of them, but *forms defined by these theories themselves* (which, he continues, implies that the axioms of formal theories are not axioms in the proper sense, i.e. fundamental unproved truths, but *definitions*). The concept of truth, according to Grassmann, also changes: in material theories truth is correspondence with the facts; in formal theories it means simply consistency (Michael Crowe's book *A History of Vector Analysis*, Dover, 1994, contains a detailed exposition of Grassmann's *Ausdehnungslehre*). The similarity with some of Husserl's (and also Hilbert's) fundamental conceptions concerning the nature of formal mathematics is striking. But, unlike Husserl, who locates formal theories not in mathematics proper, but in formal logic (formal ontology, to be precise), Grassmann does the opposite, reserving the term "mathematics" to formal mathematics and denying physical geometry, mechanics and like material theories mathematical status.

[9] As we shall see, it is the *use* of purely formal mathematical theories that Husserl saw as a possible source of philosophical problems, if not kept under surveillance so as to avoid degeneracy into a form of "technique" void of intuitive content and alienated from life.

[10] Basic principles, such as the principle of correspondence and the technique of quantization, are based mostly on formal analogies.

purely mathematical probability wave whose frequency was associated with the electron's energy (and whose velocity was *higher* than the velocity of light) could suggest to the concerned observer (as Husserl surely was) that science had lost completely the touch with reality, the reality effectively experienced with the senses, in favor of purely mathematical reconstructions of it (Heisenberg's uncertainty principle or the loss of causality and determinacy in the atomic scale could nothing but reinforce this perception).

Husserl was certainly informed of these new trends and one might believe that he thought the "Galilean" mathematical substruction of Nature had gone too far in quantum theory.[11] But this belief cannot be sustained. Although Husserl evidently noticed the obvious differences between the old and the new physics, he did not think quantum theory *in particular* stirred any crisis in the physical sciences; the problem, he thought, could not be imputed exclusively to contemporary science.[12]

What Husserl meant, in fact, was not a crisis *in science*, but *of science*; it was the *project* of a mathematical science of Nature that, he thought, was facing a crisis, not in its methods, results, or practical relevance, but in its lack of self-consciousness, by being oblivious of the sources and scope of validity of its concepts and methods. According to Husserl, the winds of crisis had been blowing for some time already – since the first days of modern science, to be more precise – and were stirred by, basically, an alienated conception of Nature and the uncritical use of formal mathematical methods (with the emphasis on "uncritical"). This undesirable situation, he thought, could only be surmounted by philosophy; not any philosophy, however, but the kind of philosophy he advocated,

[11] In truth, the mathematical treatment of quantum phenomena does not fit the model Husserl presented for classical physics: experience → mathematical modeling of experience (via abstraction and idealization) → mathematical investigation (of the properties of these mathematical models) → explanation (of past experiences) and prevision (of new experiences). The second step is missing. However, since the classical approach was, for him, already crying out for philosophical clarification; and since his theme was not the applicability of mathematics to science in general, but a critique of the scientific conception of Nature and the mathematical methods of science, he did not get much involved with the problem quantum mechanics poses for the philosophical investigation of the scientific applicability of mathematics in general, even though it also poses problems for *his* account of the applicability of mathematics to science.

[12] It is important to notice that Husserl was not an enemy of science, modern or traditional, rather the opposite; he thought the mathematical sciences of Nature were admirable endeavors, needing only some doses of philosophical criticism in order to be properly understood and circumscribed to their rightful domain of validity (but, unfortunately, Husserl criticism of "technization" will lead to Heidegger criticism of "technique" and eventually to a general pos-modernist aversion to science).

in which the very idea of an objective, transcendent and completely determined in itself physical Nature that speaks the language of mathematics is traced back to its origins in transcendentally reduced consciousness,[13] and our intuitions and "living experiences", due to their privileged epistemological status, set the boundary conditions for the rightful uses and scope of validity of formalistic thinking in science. In short, for Husserl, both the diagnosis of a crisis of science and the prophylaxes suggested served the goal of establishing phenomenology as a first philosophy that could work for a renewal of humanity based on *personal* responsibility.

As I have just said, for Husserl, this crisis was innate to the modern mathematical science of physical Nature whose creation he attributed to Galileo and other scientists of that period. The problem, he thought, did not lie in treating Nature mathematically, but in the following related points: 1) the obliteration of the intentional acts of idealization that allowed this, and, consequently, the naïve attitude of taking as an independent reality what is intentionally constituted, namely, idealized Nature, an entity that *by constitution* escaped apprehension by our senses in a complete and satisfactory way (a form of Platonism that dislodges the direct intuitive access to Nature from the fundamental position and foundational role Husserl thought it had by right), 2) the enthronization of mathematical methods as the only truly scientific ones, to be extended to any intellectual endeavor we call scientific, such as psychology or phenomenology, if phenomenology, as Husserl wanted, maintained its pretensions to scientific *exactitude*, and 3) the uncritical use of formal mathematics, i.e. a scientific methodology demanding critical assessment.

For Husserl, only transcendental phenomenology could deal with these problems, clarifying the intentional constitution of idealized (or mathematized) Nature and determining the domain of legitimacy of the mathematical methods of the science of Nature and their scope[14] (what might entail, for instance, that what is "left out" in the idealization process, what is given *directly* in experience may be precisely what interests us in other areas of science such as psychology, whose

[13] [...] a meticulous intentional analysis, strictly free of prejudices and in absolute evidence [...] does not deprive in the least the natural conception of the world, that of daily life and also of the exact science of Nature, of its sense, but proceeds to the lecture of what is effectively and properly contained in this sense." (Appendix I) All the translations into English of quotes from Husserl are mine.

[14] [...] it is not the case of dominating technically and practically the method as a "qualified worker" in the field of activity of the physicist, but to understand, by a regression to the ways of thinking of the creators of the method, including their mutations, its proper sense and rightful limits." (Appendix I)

foundations, as we know, Husserl thought as one of the main goals of phenomenology).[15] By limiting the scope of mathematics in science in general, Husserl, among other things, opened the road for *non-mathematical*, purely descriptive sciences that wanted to remain faithful to the intuitively given *as the fundamentally given*, such as phenomenology itself. Remember that phenomenology, conceived as a pure science of essences given in eidetic intuition, is out of the reach of mathematical methods (see, for instance, *Ideen* §71 and those following). In short, Husserl naturally saw a critique of the uses of mathematical methods in science, their justification and delimitation, as a much needed prelude to phenomenology understood as a non-mathematical, albeit *scientific* first philosophy.[16]

My aim here is to analyze Husserl's criticism of the mathematical science of Nature, and see to what extent it is well-founded. More specifically, I want to ask whether Husserl's recipe for overcoming the "crisis" of science can be followed in face of the specificities of scientific methodology. Also, I want to access the correctness of Husserl's model for the applicability of mathematics in the physical science;[17] i.e. whether abstracting and idealizing from perceptual experience can *always* provide mathematical models for science; whether the genesis of these

15 "The present situation in Europe, of general collapse of spiritual humanity does not change anything in the results of the science of Nature; and these results, in their autonomous truth, do not contain any reason for the reform of the sciences of Nature. If such reasons exist, they have to do with the relation of these truths with scientific and extra-scientific humanity and spiritual life. Its is the psychic and the spiritual in general, in their collapse, that forces us to create an effective and authentic psychology that makes human existence, personal existence, personal life, personal actions and their spiritual consequences, and the personal community that is built in these actions and consequences finally comprehensible, and then make us see the edifice of a new humanity." (Appendix I)

16 For Husserl, even some physical sciences, such as biophysics, cannot be mathematized in the same way physics is: "Mathematical physics is an extraordinary instrument of knowledge of the world in which we live effectively – Nature –, which maintains always in all its changes a concrete and empirical identical unity. It makes a physical technique practically possible. But it has its limits; not in the fact that we do not, empirically, leave the level of approximations, but in the fact that it is only a narrow layer of the concrete world that is in this way effectively grasped. Physiology, biophysics, as doctrines of organic bodies in the totality of the concrete organic world, can borrow from physics as often as they want (organisms being in fact able to be idealized as mathematical bodies); it remains true that, as a principle, biophysics cannot ever dissolve into physics. Biophysical reality and causality cannot ever reduce to physical reality and causality." (Appendix IV). Needless to point out that the development of the biological sciences denies emphatically these assertions.

17 There are two senses in which we can say Husserl identifies an "application" of mathematics to the sciences of Nature; the first in the construction of mathematical models of Nature via idealization; the second, in the use of mathematics for the investigation of these models (which, as Husserl claimed, we tend to erroneously see as inquiring into Nature *itself*, not only ideal *models* of Nature).

models, whenever available, can *always* be traced back to the life-world; and whether there is any scientific reason for somehow restricting the use of *purely formal* mathematical methods in science.[18] If we answer these questions negatively, the project of avoiding formalistic "degeneracy" in the mathematical sciences of Nature by rooting them in the *Lebenswelt* will be, of course, severely limited.[19]

So, let us begin by seeing how Husserl thought Nature – the Nature we experience with our senses – becomes a collection of more or less articulated mathematical manifolds:

1) First step, idealization; i.e. moving from the intuitively given to ideal reconstructions of it. For instance, magnitudes such as time, distance, velocity or temperature, which are, and can only be experienced as *discrete* sets of values mathematically represented by *rational* numbers, are thought, within some range, as *de facto* continuous magnitudes whose values can only be accurately represented by *real* numbers. Idealization here amounts to taking observed magnitudes as *actually* continuous, and their values as *actually* represented by real numbers, not simply the rational values we observe in measurements. Idealization proceeds with the identification of all the *possible* values of physical magnitudes with sets of mathematical entities (often, but not always, real numbers), and correlations among the values of these magnitudes as *mathematical functions* defined on these sets. These are usually the first (but by far not the only) moves of mathematical idealization in physics. Many others concur and superpose: bodies are reduced to massive *points*, trajectories to perfect geometrical *lines*, physical space to *geometrical* space, interactions among bodies (gravitational, electric or magnetic forces, for instance) to *mathematical fields*, dynamic processes to *differential equations*, etc, etc.[20]

18 My arguments will have no bearing on Husserl's project of restricting the use of formal methods in science for the sake of "spiritual renewal", but they will, however, make it clear that this restriction cannot be justified *methodologically*. I plan to show that, from a strictly scientific perspective, Husserl is wrong in believing that formal methods must always be ready to be instilled with their "original meaning" in order to be adequate and safe. More, I think Husserl's criticism is based on a misconception regarding the nature of mathematics and modern physics, and if scientific methods must for non-scientific reasons be restricted, the development of science will pay the price.

19 But I will not touch the question whether Husserl is right or not in seeing the "formalistic alienation" of modern science as part of the "crisis" of meaning of modernity (I particularly do not think it is).

20 "To the vague notions of larger and smaller, of more and less, and vague identity, there can be substituted with all their determinations the exact "how many", the exact "how bigger" and the exact "how smaller", or still the exact identity." (Annex I). Idealization is for Husserl, an activity of *reason*: "[...] logical concepts are not concepts extracted from the simplicity of the intuitive; they grow by an activity proper to reason: the formation of ideas, the exact formation of

The next step is, for Husserl, the philosophically suspicious misstep. The reality we *actually experience* is degraded to the condition of only an imperfect approximation to the world of idealized entities we substituted for it. Mathematical manifolds of ideal entities obtained by "exactification"[21] take the place of aspects of the world *we experience* as the *truly real* aspects of the world.[22] Mathematical idealities, in Husserl's jargon, provide a "hypothetical substruction" of the world we experience. The substruction is hypothetical in the sense that it is established *ex hypothesis*, but also in the sense that it must be submitted to experimental confirmation. This, however, can never be definitive, for *ex hypothesis* experience can only approximate true reality. Experimentation is then "contaminated" by the very hypothesis it is called to confirm.[23]

In short, for Husserl, as far as the applicability of mathematics is concerned, the first task of mathematics in science is to provide models of Nature, obtainable by a combined process of abstraction ("focusing" on particular aspects of the given; usually formal aspects, which makes the process one of *formal* abstraction) and idealization (exactification). Husserl's criticism at this point is directed mainly at the reversal of priority that science, he thinks, operates: mathematized Nature taking the place of intuited Nature as the truly basic reality.

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concepts; for example, by this idealization that, in opposition to vague empirical lines and curves, produces the geometrical line, the geometrical circle." (Annex I)

21 "How can true mathematical Nature be determined from normal appearances [*normal appearances = data of normal sensibility in relation to normal understanding, JJS*]? This happens by means of methods of 'exactification' of continua, of sensible causalities into mathematical causalities, etc." (Annex I).

22 "'Mathematics and the mathematical science of Nature', or still the *dressing with symbols* of symbolic-mathematical theories, contain all that which for the expert and the cultivated men replaces (as the objectively real and true Nature) the life-world, substituting it. It is this covering of ideas that makes us take for the true Being what is only a method – a method that is there to correct, in an infinite progression, by "scientific" anticipations, the "rough" anticipations that are originally the only ones that are possible in the realm of the effectively (really and possibly) experienced in the life-world. It is this covering of ideas that renders the authentic sense of the method, formulas and theories incomprehensible, and that, due to the naiveté of the method at its birth, was never understood." (*Crisis*, 9h)

23 "[The mathematical physicists] prepare the forms of hypothesis that will be the only acceptable as hypothetical possibilities for the interpretations of the causal regularities that the continuous progression of observation and experimentation will observe empirically; interpretations that connect them to ideal poles and their exact laws. The experimental physicists are also, in their task, constantly oriented towards ideal poles, numerical magnitudes, general formulas. Those remain, then, in all scientific research, at the center of interest. All the findings of old and new physics took place, if we can say so, in the world of formulas coordinated with Nature." (*Crisis*, 9g)

I can sympathize with Husserl with respect to this last point – mathematical Nature is definitively not real Nature – but, at the same time, I think Husserl fails to see that it is not always the case that mathematical models of Nature are idealized abstractions from the intuitively given. Think of phenomena in atomic scale, notoriously difficult to access intuitively. Often physicists have to rely on mathematical constructs (for instance, the already mentioned wave of Planck's) that have nothing to do with abstracting and idealizing from experience; and as often as not, quantum physicists must do without any models at all (as, for example, the already mentioned mathematization of quantum mechanics in terms of matrices). But, to be fair, classical physics follows largely (but not completely) the pattern Husserl traced (which, however, would make contemporary science impossible if strictly enforced. I will come back to this later).

2) Then comes, according to Husserl, a decisive move: the mathematical coordination of mathematical idealities by means of mathematical formulae is taken as the very essence of reality. So, by commanding mathematical models of reality, formulas are believed to command reality itself. All possible intuitions (which, remember, are always conceived only as approximations) are taken as a priori *determined* by formulas (within a certain range of acuity, of course, since "true" – i.e. mathematical – reality always eludes us): we can predetermine *mathematically* the outcome of experiments. Nature is now completely submitted to mathematics, for only mathematics can give *accurate* access to Nature (as opposed to the approximations granted to our senses). Husserl sees here another reversal of priority: even though mathematics is "nothing but [only] a particular practice" in the world, mathematics is thought to command the world. Although mathematics has its genesis in some practices of our lives, he thinks, mathematics directs our lives.[24] He says:

> One is in possession of *formulas*; one possesses then, simultaneously, a priori, the desired practical *anticipation* of that which is to be expected in the concrete real life that follows with empirical certitude in the world of intuition – the real life where mathematics is nothing but a particular praxis. The decisive operation for life is then mathematization, with the formulas at whose elaboration it aims (*Crisis*, 9f).

If we put (1) and (2) together we get that, for Husserl, the mathematical science of Nature begins with abstraction and idealization, when aspects of the Na-

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24 This observation contains, in a slightly reformulated version, the central question concerning the philosophical problem of the applicability of mathematics to the natural sciences: how is it possible that mathematics, *a product of free human creativity*, can have anything to say about the natural world out there, existing *independently* of us?

ture we experience are modeled by mathematical manifolds, and then proceeds by submitting mathematized Nature to the scrutiny of mathematical theories (even purely formal mathematical theories – in which case, he thinks, strict surveillance must be exerted. I will come back to this later), which dominate the word of experience by dominating the mathematical manifolds we take for Nature itself. One of the main goals of mathematical theories of Nature – besides revealing the architecture and functioning of the machinery of the world, since science "naïvely" presupposes that the mathematical models of the world *are* simply the world – is to establish formulas, equations and the like from where anticipations of experience can be extracted. In few words, for Husserl, "Galilean" methodology consists basically in moving from immediate experience – the realm of intuitions – into the formal, onto which is laid the burden of regulating by means of its formulas the whole field of possible intuitions.

A question now presses itself: which "formulations", i.e. which mathematical theories can *rightfully* take upon themselves the honorable task of *determining* with their formulae the entire field of possible experiences? Consider the following quote:

> [T]he passage from the mathematics of real domains to its logical formalization and the consolidation of the autonomy of extended formal logic understood as pure *Analysis* (or pure theory of manifolds) are in themselves something *correct*, and also necessary; as well as the "technization", with the total immersion into thinking that is purely technical that belongs to it. But all this can and must be a method understood and used *in a fully conscious way*. This is not the case, however, if one does not constantly avoid, in using it, dangerous *changes of meaning*; that is, provided that the original sense-bestowing of the method, from where it derives its sense as accomplishing the knowledge of the world, stays always actually accessible; also, moreover, provided that it is free of all *traditionalism taken for granted*, traditionalism that already in the original discover of the new idea and the new method allowed moments of obscurity to mix with meaningfulness. (*Crisis*, 9g).

So, although Husserl admits that *purely formal* mathematical theories are bona fide *logical* theories, he is not willing to admit them into mathematical physics *if certain precautions are not taken.* For him, the reliance on purely formal methods – which implies necessarily some degree of "technization"[25] – are acceptable, pro-

[25] Husserl defines "technization" thus: a "mutation of a thought that experiments, discovers, creates constructive theories (sometimes of the highest geniality) into a thought whose concepts have experienced a mutation, becoming "symbolic" concepts." (*Crisis*, 9g) In short, technization means symbolization and knowledge by symbolic manipulations devoid of meaning.

vided that, firstly, we do not loose from sight that it is *this world we live in and intuitively experience* that we want to know, which translates into the demand that the *meaning* of the symbols and the operations with them can be at any moment, at least in principle, recaptured by reactivating the original sense-bestowing acts. And secondly, we do not allow "moments of obscurity" to come in (as, we can conjecture based on Husserl's treatment of the problem of "imaginaries" in mathematics, happened when imaginary numbers were introduced and used as mere technical instruments without the proper understanding of *why* they could be so used).

So, Husserl's demands are very strict: the theories we set in action in order to investigate the mathematical models of Nature, and then, derivatively, Nature itself, even if transformed into a "technique" for practical reasons, must be ready, if necessary, to be instilled with their original meaning upon reactivation; and, most importantly, imaginary entities (i.e. entities that are not abstracted and idealized from the intuitively given), if introduced, cannot bring in "obscurities" (we will soon see how Husserl thought this can be avoided).

There is however one question Husserl does not ask, and which is fundamental for a *correct* assessment of the role of mathematics in science: do the idealized mathematical manifolds we substitute for Nature *impose* their own theories, or do they offer themselves to the most convenient ones? In other words, are we *free to choose* the more adequate mathematical theories for doing mathematical physics? The reason why he does not raise this question is that he already knows the answer: we are not; our mathematical models have their own, privileged theories, which must be extracted *intuitively* from these models themselves.

But, as Husserl certainly knew, scientists are almost never content to remain within the boundaries of these minimal theories. This is, after all, typical of mathematics; mathematical theories are often extended into other mathematical theories and mathematical domains into other mathematical domains. Once an aspect of Nature has been mathematized it shares the fate of any mathematical manifold: to be up for grabs, for there is no a priori restriction on which mathematical theory can be summoned to take care of it, the theory intrinsic to the manifold (whose language, concepts and basic truths are offered directly in intuition), any theory whose domain is obtained from the original manifold by the adjunction of new (theoretical, "imaginary") entities, or even any theory only formally identical to any of the above.

But Husserl would not accept such liberalism. The imperative that scientists should not loose the world of our experience from sight requires in the first place, as we have seen, that the mathematical models of experience be obtained *from experience*, and then, that we be careful not to submit these models to shifts of meaning (supposedly by immersing them into other manifolds or extending them by the adjunction of new "imaginary" elements, when, Husserl thinks, they loose their sense as models of *this* world).

For Husserl, there are privileged theories in mathematical physics, namely, those extracted from the idealized models of Nature (the mathematical manifolds we substitute for it) by mathematical intuition. I will call them *natural theories*. Their meaning cannot be jeopardized on pain of loosing science's precious link with the "living experience". But, as even Husserl had to admit, mathematical theories often dialogue. So, which theories he thought should be allowed to converse with the privileged natural theories? Can natural domains be put under the care of mathematical theories that are not their own, even purely formal theories? From what Husserl explicitly said, we can conclude that he accepted some mingling of theories, but under strict conditions.[26] Well, then, which conditions?

Here reemerges a problem that had already occupied Husserl in his first book, *The Philosophy of Arithmetic* (1891) and other texts of that and later periods (the Göttingen Lectures of 1901, in particular): how can symbolic manipulations tell us what the facts are in the domains of materially determinate theories? In our case, how can playing with symbols according to rules can be given the *right* to tell what the facts are in the mathematical models of Nature, and then, indirectly, in Nature itself, anticipating the (ideal) outcome of experience?

Husserl answers: provided symbolic manipulations are either carried out within symbolic theories *isomorphic* with natural theory, or else *natural theories are logically complete* (i.e., *natural manifolds are definite*) and symbolic manipulations are carried out in consistent symbolic extensions of them. The first, although not *explicitly* mentioned in *Crisis*, is the obvious generalization of the justification he presented for symbolic arithmetic in *PA*; the second is his general "solution" to the problem of imaginaries in mathematics, and is explicitly mentioned in *Crisis*. There is, then, a common tread linking Husserl's first and last published works, *PA* and *Crisis*, respectively: some reservations concerning "purely" symbolic methods of knowing. It surfaced in *PA* under the guise of the need for a logical-epistemological justification of symbolic arithmetic; in *Crisis*, in the need of a justification for the technical methods of the mathematical sciences of Nature, and

[26] With respect to algebraic symbolic reasoning Husserl says: "[...] the powerful elaboration of signs and ways of algebraic thinking, a decisive moment which was, in a sense, pregnant with future fruits, and in another disturbing for our fate." (*Crisis*, 9f) He is here alluding to the methodological relevance of symbolization and, at the same time, the risk of alienation it poses (the "crisis" for science it potentially carries).

the task of overcoming their "formalistic alienation" by a philosophical analysis of the genesis and scope of these methods.

But are these restrictions reasonable; did Husserl really understand the *unrestricted* formal nature of mathematics (even materially determined mathematics like physical geometry) and the role of mathematics in science? Let us examine this question. Mathematics is a formal science, meaning that mathematical truths are invariant under isomorphisms (i.e. mathematical truths are *formal*). Being formal there is no reason why a domain cannot be investigated by any theory whose domain is isomorphic to it. But, obvious as this fact may be, Husserl never affirmed it with its entire letters (*even though it is the formal nature of contentual arithmetic that is behind his strategy of justification for symbolic arithmetic in PA*). He never explicitly said that *the only thing* mathematics – even contentual mathematics – cares about are the formal (or structural) properties of its domains, not the objects these domains contain or the concepts governing them. He believed that, although formal, mathematical theories are, or should eventually be, when *scientifically* relevant, theories of *determinate* objectual domain or, indifferently, *determinate* concepts. Science, real science, Husserl thought, is *always* of objects or concepts, even if the *only* aspects of its domain some particular science (like mathematics)[27] can ever know are those exclusively formal.

Husserl managed to justify symbolic arithmetic epistemologically by showing that there is an isomorphism between its formal domain and the domain of conceptual arithmetic; i.e. in the fact that the domain of numerical concepts and conceptual operations is *structurally identical* with the domain of symbolic representations of numerical concepts and operations with these symbols. But Husserl missed in *PA* the opportunity of asking the fundamental question: *why* can we obtain mathematical knowledge of a domain by investigating another isomorphic to it? The fact *that* this is possible satisfied him. Husserl seemed to think that by manipulating numerical signs we are still, indirectly, manipulating numbers, as a puppeteer moving his puppets. But the truth, of course, is that moving from numbers and operations with them to signs and symbolic operations is useful and safe *only because the structures of the numerical and the symbolic domains are identical*; what *really* interests *contentual* arithmetic is indifferently instantiated in any domain isomorphic to the numerical domain.

Had Husserl drawn all the consequences from this he would have realized that, as far as mathematical interests are concerned, moving to the level of the

[27] Remember that Husserl distinguishes between mathematical *science*, such as physical geometry, and applied mathematics in general, from purely formal mathematics, which is for him a chapter of formal ontology, and thus pure formal logic.

purely symbolic is not necessarily tantamount to leaving behind the level of intuition, for the simplest fact that the *same* structure can manifest itself in either level. If formal properties can manifest themselves as properties of meaningful mathematical entities (denoting something in Nature), so they can when these entities are substituted by any other objects, even mere symbols, provided the subjacent structure is preserved.[28] Present or absent, mathematical entities do not really matter; only the formal properties of their arrangement are of interest.

Husserl's justification for the use of symbolic methods in general – when there is maybe no isomorphism between the domain of interest and the symbolic domain – required that the theories to which we bring symbolic help should be

[28] I use the terms "structural" and "formal" interchangeably because structural properties are formal. We could define mathematical structures simply as the objective correlates of mathematical theories. It is in this way that we say, for instance, that group theory characterizes the structure of group. Two structures are, in this sense, *equal* if they are correlated to logically equivalent theories. In this sense, a structure has no property its theory or any theory equivalent to it cannot show. This poses a problem if the theory is not logically (syntactically) complete; structures would then be in general only partially determined. We can remedy this by saying that theories in fact only characterize *families* of structures. The problem would then remain of characterizing single structures. We could do this by means of *complete* theories, but, even though all the properties of a structure would in this case be determined in principle, this definition would make the notion of structure depend on the logical powers of the underlying logic or the expressive powers of the language we choose. In order to make the notion of structure a purely semantic notion it seems more convenient to adopt the following definition, more mathematical than logical: mathematical structures are the common aspects of *isomorphic* mathematical domains. We can say this is the *abstract* (or semantic) notion of structure, as opposed to the *theoretical* (or syntactic) notion just discussed. In this sense, structures can be characterized by *categorical* theories. Given a categorical theory **T**, the structure it characterizes is the structure of its models (since they are all isomorphic). Any theorem of **T** establishes a formal property of the structure **T** characterizes, but there may be properties of this structure that are not theorems of **T**, if **T** is not complete (not all categorical theories are complete). In particular, the numerical structure is the formal ω-sequence, characterized by second-order arithmetic (which, however, cannot prove everything that is true of it). The interesting aspect of this notion of structure is that, in order to prove no matter which fact about a structure, we are not confined to a particular domain that instantiates it or a particular theory that characterizes it. Husserl had his own definition of structure (which he calls *formal domains*). He defined them as intentional (objective) correlates of formal (i.e. non-interpreted) theories, i.e. structures in the syntactic sense. For him, a formal domain could always be completely characterized, if not by its theory, by its *complete* extension, which he thought could always be obtained (he wrote these things decades before Gödel). It follows from Husserl's definition that a formal theory always determines a *unique* formal domain, that any theorem of a formal theory is true in the formal domain it defines, and that all (formal) properties of a formal domain can (ideally) be derived in its theory, since it is (ideally) complete. So, there is no clear distinction for formal domains between the semantic and the syntactic notions of truth; therefore, no clear distinction either between the syntactic and the semantic notions of completeness.

logically complete, a condition he thought to be universally reachable in principle. Husserl's injunction that a theory should be "master of its domain", meaning by it that it should be logically complete, i.e. capable in principle of deciding any question that it could raise concerning its domain, translates his firm believe that mathematical theories are theories of *specific* domains (which, of course, are thought as completely determined "in themselves"). This is also true of purely formal theories, which, according to Husserl, also have their own (formal) domains. But, of course, since mathematical theories, even interpreted theories, are invariably formal (their truths are invariant under isomorphisms), there would be no problem in investigating the formal properties of a given domain **D** in the context of any theory **T** whose domain extended **D**, i.e. whose domain admitted a sub-domain **D'** isomorphic to **D**, *even if **T** were a purely formal theory*. If we were able to show in **T** that φ was a property valid for the elements of **D'**, then, being formal, φ would also be true of **D**.

This all too common mathematical procedure (for instance, one can show that a *real* equation of the third degree has a *real* solution by operating with *imaginary* numbers) is strongly limited by Husserl, for he thinks that, being "masters of their domains", theories can only get a helping hand from fellow theories (previously stripped of their material content and reduced to their purely formal framework) if they do not really need it.[29] Other theories may provide more con-

[29] Here are some examples of theories that are not "master of their domains", and gain with it:

(1) The introduction of points and the line at infinity in the Euclidian plane (merely formal objects without any material meaning, given that they do not correspond, as idealizations, to nothing we can experience) by Kepler, Desargues, Pascal, La Hire and Poncelet and other creators of Projective Geometry brought about a formal mathematical theory that proved its utility by proving, by new methods, important theorems of Euclidian Geometry involving properties preserved under projections. It also made it possible the introduction of important methodological principles such as the Principle of Continuity (that allowed for a uniform treatment of the conics, in which all conics are seen as projective images of the circle) and the immensely useful and elegant Principle of Duality. (In a typical mathematical way, we can also treat standard problems of Projective Geometry by other mathematical means, for instance, vectors in the complex plane – see, Nikolic', Aleksander M., "Karamata's Products of Two Complex Numbers", *The Teaching of Mathematics*, vol. VII, 2, pp. 107-116. 2004.)

(2) The introduction of negative and imaginary (complex) numbers into arithmetic by the Italian algebraists of the 16th century made it possible a uniform treatment of algebraic equations.

(3) The creation of infinitesimal methods in geometry by, for instance, Cavalieri, allowed for an improvement over the Archimedean method of exhaustion. Whereas Archimedes' method was one of justification rather than discovery – indirect justification, to make things worse – infinitesimal methods were of both discovery and justification – *direct* justification, to make things better. The apparent "problem" that infinitesimals are not, and could not be geometrical entitie

venient, but not more powerful methods (in the sense of methods able to prove more results). For Husserl, theories should be ideally complete because all they need in order to decide any question that can be asked concerning their domains should come *intuitively* (directly or indirectly) from these domains themselves. The primacy of the intuitively given – and then of the I – over the purely symbolic that conditions the solution to the problem of imaginaries in mathematics Husserl presented in the beginning of his philosophical career is still clearly noticeable in the strategy he proposed for bringing science back from alienation and naïveté into full clarity concerning sense and methods.[30]

But, again, why should the formal aspects of a given mathematical manifold be revealed only by investigating this manifold and not any other having a subdomain isomorphic with it (and so, with the *same* formal properties of the original domain)? If we abandon Husserl's prejudice, for it is really a prejudice, that science is always science of *determinate* things (objects, concepts, essences, etc) and accept that it can also be of nothing in particular, or many different things having the same identical formal properties, or, if *formal structures* are included in the list of things we can have sciences of, then the restriction of completeness loose relevance. There is no need for a mathematical theory to be "master of its domain" if this domain, with respect to its mathematical properties, can also be investigated by other, maybe more resourceful theories.

As I said above, in *Crisis* Husserl did not open the full spectrum of problems concerning the applicability of mathematics to natural science. In particular, he took for granted that it is applicable (as a conceptual and symbolic system suitable for the modeling, upon idealization, of our experience, and as a provider of

properly speaking, but only "mere" formal objects did not seem to bother so much the creators of the method.

[30] "The sense of being of the world given in advance in life is a *subjective formation*; it is the work of life – pre-scientific life – in its experiencing. It is in this life that the sense and validity of the being of the world is built; that is, always of *this* world that is at any time effectively valid for the subject of experience. With respect to the "objectively true" world, that of science, it is a *formation of a superior degree*, whose foundations lie in the pre-scientific thinking and experiencing and their operations of validation. Only a radical regressive inquiry on subjectivity, I mean the subjectivity that renders ultimately possible all validation of the world with its content, in all its scientific and pre-scientific modalities, an inquiry that considers the whats and hows of rational performances, can render comprehensible the objective truth and attain the *ultimate sense of being* of the world. Then, it is not the being of the world in its unquestioned evidence that is in itself what exists primarily, and it does not suffice to pose simply the question of what belongs to it objectively; *on the contrary, what is primarily in itself is subjectivity*, and it is *as such* that it pre-gives naively the being of the world, and then rationalizes, or, what amounts to the same, objectifies it." (*Crisis*, 14)

theories for the mathematical models of experience), but did not question why.[31] Also, as I have just argued, given Husserl's strategy of justification of standard mathematical methods, it is clear that he thought natural manifolds should be "definite", meaning that they should be defined by a complete system of axioms, corresponding to the "logical-formal idea of a world in general"[32] (*Crisis*, 9g). Of course, if a definite manifold corresponds to the idea of the logical form of a world in general, the world itself, considered in its formal (abstracted, idealized) aspects, must ideally constitute a coherent system of definite mathematical manifolds.

This may stand as an ideal for science, but since it is far from realized, and there is no guarantee that it could be realized (in fact, if we consider that logical-formal axiomatic systems must be designed according to certain reasonable effectiveness restrictions, Gödel showed that it is in general unrealizable), the mathematical science of Nature must do as it can, just like mathematics itself, which is to freely extend domains and theories one into the other in order to obtain formal truths that can be transferred from one domain to any other formally equivalent with it.

I must now, to conclude, consider the intentional genesis of natural manifolds on the basis of perceptual intuition (considerations of this kind fall within the scope of genetic phenomenology).[33] The question I want to ask is whether

[31] The only philosophical problems related to the applications of mathematics to science Husserl recognized can be solved, he thought, by *restricting* the applicability of mathematics in science. His strategy is *restrictive*, rather than *explicative*.

[32] "The presupposition of classical physics: the subjectively changing natural phenomena, the empirical phenomena with their empirical progress in terms of worse or better approximation (their perfecting), points towards the mathematical idea of a Nature in itself as a unified world of bodies in themselves. This implies a universally valid mathematics of Nature [... and in it] lies the foundation of a causal legality, according to which all bodies, whose ideal essence consists in their causal, space-temporal being, are calculable. The general mathematical legality is definite in the sense that it has the form of a finite numbers of fundamental mathematical laws (the axioms) in which all laws are included, in a purely deductive manner, as consequences." (Appendix IV).

[33] With respect to geometry, Husserl carries out such an investigation in the famous appendix III, "The Origin of Geometry". But the general task of a phenomenological critique of science (in the sense of a radical foundation of science in transcendental subjectivity) is much broader; it consists essentially in questioning the "evidences" belonging to a world that is already here, presupposed, with the sense it has, by science and our unquestioned being in the world. Some of these evidences (concerning the world as simply "given") are: 1) it is a coherent unity subjected to strict causality; 2) its being is completely determined; and it corresponds to the being of the world a system of truths in themselves, the possession of which constitutes the goal of objective science; 3) "the world is, and already was, here; [...] the correction of any apprehension of this world, be it an apprehension of experience of any other, presupposes already the world in

Husserl is right in believing that the mathematical models of Nature are delivered by abstraction and idealization from our immediate perceptual experience of it.

In the genetic analyses hinted at in *Crisis* (but not spelled out in details), the relation between perceptual and mathematized Nature is cast in terms of dichotomies such as concrete versus abstract, real versus ideal, intuitively given versus categorically constituted. For Husserl, the first terms of these dichotomies designate starting points, the second, points of arrival; constitution is what happens in between. The idea of a constitution of the ideal in the real reverses the order of priority in Plato's idea of participation; by adhering to it Husserl implicitly criticizes the extreme "Platonism" of modern science.[34] For him, the real world of our direct experience is not an *imperfect* copy of the ideal world of science, but *conversely*, it is the ideal world that *originates* in the real by intentional acts of consciousness (abstraction and idealization).

I will, however, skip the details of the genesis of mathematized Nature in intentional consciousness; for our purposes it suffices to notice that, for Husserl, as already stressed, mathematics comes into science primarily by way of the substitution of the given of experience by mathematical manifolds, i.e. mathematical modeling. The question I want to raise is whether this covering of the intuitively given with mathematical concepts is univocally determined by experience or, contrarily, mathematization is sub-determined by the given. Is the road from the senses to understanding a one-lane road? Is the covering of perceptual data with categorial formations without alternatives? How did Husserl answer (or would have answered these questions)?

Let us consider the geometrization of perceptual space as a paradigmatic case. It has been argued that Husserl believed the mathematization of physical space borrowed from different sources. Some very general geometrical properties (for instance, continuity, homogeneity, the character of being isotropic, the possession of a definite, although indeterminate dimensionality or the existence of a

its being as an horizon of all that is indubitably valid, which implies a certain stock of things known and certitudes subtracted to any doubt, with which to be in contradiction means to be devalued and reduced to nothing"; and 4) the experience of the world can be infinitely perfected. To trace the "genesis" of *this* sense of the world in consciousness and investigate how it acquired it constitutes the core of a phenomenological critique of the presuppositions of science (see § 28).

[34] "For Platonism, reality had a relation of 'méthexis' (participation) more or less complete in the ideal. This opened for ancient geometry some possibilities of application – primitive application – to reality. But in the *Galilean mathematization of Nature* it is Nature *itself* that, under the direction of the new mathematics, is idealized: it becomes, to employ a modern expression, a mathematical manifold." (*Crisis*, 9)

metric characterized only in its most general aspects) would simply be consequences of physical space being a particular type of Riemannian space – the "truths" expressing these "facts" about physical space would then be analytic –; other geometrical properties of space, such as the possession of a *particular* metric or a *determinate* number of dimensions would be empirical, i.e. given by the senses.[35]

But how can we *know* that physical space is a particular type of Riemannian space to begin with? Do we really *know* this or do we simply *presuppose* it? Could we know a posteriori, from the testimony of the senses, that physical space has the structure of a Riemannian manifold? Certainly not, the senses are too coarse and limited for deciding both the fine-grained and the global structural aspects of space. Not being a posteriori, or at least not completely a posteriori, this knowledge must be to a large extent a priori, and so Husserlian constitution of idealized space would come close to a form of Kantian-like pure intuition of spatial features: a *particular* course for the process of idealization would appear as *necessarily* induced by experience. In fact, this is how Husserl seems to understand idealization; at least as far as geometrical idealization is concerned. For him, to idealize is to "pass to the limit", "to follow our noses" along the perceptually given to its *necessary* perfecting in one, but not another way.[36]

If Husserl really believed that the general Riemannian character of perceptual space was essentially a matter of right, belonging to the *essence* of perceptual space – and what makes it the particular Riemann space it is, Euclidian, three dimensional, etc., would be a matter of fact – then he cannot be right simply because a Riemannian manifold is not the most general spatial structure one can conceive. For instance, we can conceive of a space that is not locally flat everywhere as presupposed by Riemann (a presupposition associated with the line element being given by the square root of a second order differential expression). Since what makes our perceptual space Riemannian cannot be a matter of either necessity or fact, it must be a priori in another, "non-transcendental" sense. This can only be what Poincaré wanted it to be; the mathematization of perceptual space is a matter of free choice, guided more by convenience than (either necessary or contingent) truth.

But Husserl's analyses of the intentional genesis of geometry in "The Origin of Geometry" seem so convincing that we are easily lead to believe that the road

[35] See Guillermo Rosado Haddock's review of Elisabeth Schuhmann's edition of Husserl's lessons of 1908-09, *Husserl Studien* (2008) 24:141-148.

[36] Of course, free variation can lead us into many different ways and produce a variety of results, but free variation has to do only with *possibilities*, not *realities*.

from our direct experience of space to its mathematical models, if not a one-lane road, is strongly biased by the "immediately given". What Husserl failed to see is that our spatial intuitions are from the start *contaminated* by presuppositions that are *not* necessary.[37] Our perceptual experience of space is already dressed with categorial formations, and if by becoming conscious of them, we remove them, what remains is in general willing to accept different and often contradictory categorial structuring. Physical space as experienced, free of any presuppositions, can be idealized as finite or infinite, continuous or discrete, Euclidian or non-Euclidian, Riemannian or non-Riemannian; experience alone cannot predetermine its "correct" mathematization.

We can, I think, generalize this conclusion, for in general our perceptual experiences are far too meager to disclose any necessary overall structure. In physics we often have to deal with realms of experience, such as the quantum domain, to which we have very limited intuitive access. In cases such as this the mathematical science of Nature must basically look for whatever mathematical patterns it can fruitfully impose on the mass of row empirical data, with virtually no extra intuitive hint as to subjacent structure; any physical or mathematical model, or even no model at all, just any theory able to organize the data and make correct previsions will be a strong candidate for the right theory, independently of any, often missing, intuitive guidance.

The most important fact concerning the questions I have been treating here is that intuition is amenable to mathematical treatment – i.e., mathematized – provided it is *only* considered in its structural or formal aspects,[38] since mathematics does not care for material content, but only for form: mathematical *is*, after all, a *formal* science. Mathematical modeling consists then in *choosing* the mathematical formal manifold that adequately expresses the formal aspects we discern in experience, which always leaves considerable room for alternatives.[39]

[37] As Helmholtz noticed, our perception of space is from the start contaminated by certain formal *presuppositions* – namely, that rigid bodies are free to move, anywhere and in any direction, without deformations – which make Euclidian geometry seem the "natural" geometry of physical space. Notice that this particular "truth" is neither analytic (although, as a presupposition, it is a priori), nor empirical. By exclusion, it must then be synthetic a priori. See "On the Factual Foundations of Geometry" and "The Origin and Meaning of Geometrical Axioms", in *Beyond Geometry: Classical Papers from Riemann to Einstein*. New York: Dover, 2007.

[38] James Jeans says that, in physics, we "must limit ourselves to describing the *pattern of events* [my emphasis] in mathematical terms", that "one may dig, one may sow, one may rip. But the final harvest will always be a sheaf of mathematical formulae." (*Physics and Philosophy*, New York; Dover, 1981, p.15)

But the way Husserl talks of mathematical concepts (such as those of geometry) as limit points of empirical (or morphological) concepts (the circle as the ideal limit of the merely circular, for instance) seems to suggest that maybe in some cases (for instance, Euclidian physical geometry as the *necessary* limit point of a process of abstraction and idealization from plain perception) experience does indeed condition the ideas we substitute for it. The fact, however, is that the process Husserl calls idealization involves to some degree choices conditioned even by pragmatic and esthetic criteria.

The hard fact about the mathematical modeling of Nature is that our direct experience of Nature provides only some initial and boundary conditions. It cannot determine the entire process. Abstracting and idealizing are not, even if not taken as univocally determining the outcome of the process, very relevant actors in our efforts to submit Nature to mathematical treatment. The combined facts that mathematics is a formal science, that physics only cares to know the formal aspects of Nature (in *this* consists the essence of Galilean methodology, in emphasizing the *how* rather than the *why*, the *mathematical form* of the mechanism of the world rather than *what* this mechanism is), and, most importantly, that there is *no a priori* restriction on which mathematical theory can be mobilized so as to better study the formal properties of Nature explain why *degrading* the role of direct perception and the process of guided abstraction and idealization can actually *foster* the progress of science and account for its immense success.

Husserl's project of "rooting" the mathematical science of Nature in the life-world, even if this would bring science back to the sphere of meaningfulness he though it had departed, can only jeopardize its development. This project seems acutely inappropriate, given in particular the strangeness of some of the realms science is called to investigate compared to the cozy familiarity of the life-world. There is no necessary link between what the life-world makes available to us and the mathematical manifolds we mobilize in the mathematical sciences of Nature.

[39] An interesting example of the adoption of a physical and, correlatively, mathematical model exclusively for the sake of deriving mathematical formal properties observed in a *different* empirical context can be found in Maxwell ("On Physical Lines of Force", 1861). Although he did not believe that his treatment of electromagnetic phenomena in terms of vortices and strains in an elastic mechanical medium corresponded to electromagnetic reality, he adopted such an approach in order "to make use of the mathematical analogies of the two problems to assist the imagination in the study of both". This is as good an example as any that for the physicist only formal properties (which Maxwell expressed as "mathematical analogies") matter (and this is why mathematics is so useful to him).

But the Husserlian model of the applicability of mathematics to the sciences of Nature, as discernable in *Crisis* points to the opposite conclusion: abstraction and idealization determining, maybe not completely, but in any case more determinately than I think reasonable to suppose, the acceptable mathematical models of intuitively given Nature, which should then be left to the care of their ideally complete theories, whose foundations should lie on an intuitive grasp of the ruling concepts and basic truths of their domains. The history of science, most notably in the last century, has shown that the use of mathematics in science did not and, most importantly, could not go the way Husserl pointed. Nor I see any reason why it should, since, as I believe to have made clear here, Husserl failed to see, in full clarity, despite his insightful remarks on the nature of mathematical knowledge, the real nature of mathematics and the pattern of its applicability to science.

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