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PHILIP KITCHER'S VIEWS ON A PRIORI KNOWLEDGE

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Traditionally, mathematics and logic are considered paradigms of disciplines constituted by a priori knowledge. In addition many sentences the content of which is neither purely logical nor purely mathematical are said to be known a priori: "All bachelors are unmarried men", "All bodies are extended" and "Nothing is simultaneously red and green all over".

In order to get clear about the property of being a priori we have to settle what the term "a priori" primarily applies to. Furthermore, any adequate account of the way in which a priori knowledge is independent of experience has to allow room for a degree of *dependence* on experience: certain experiences may be necessary to equip ourselves with the concepts needed if we are to entertain a candidate for a priori knowledge in the first place - or indeed, in the case of inferential a priori knowledge, if we are to understand the premises for the inference in question.[1]

Philip Kitcher has systematically worked on trying to provide necessary and sufficient conditions for knowledge to be a priori in his book *The Nature of Mathematical Knowledge* (1983).[2] Some philosophers of mathematics[3] have taken a de-

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[1] Basic a priori knowledge is knowledge which is not obtained by any inference from other premises. For example, elementary arithmetical truths like "2 + 2 = 4" and trivially analytic truths like "All bachelors are unmarried men" are considered items of basic a priori knowledge. In contrast, inferential a priori knowledge is knowledge obtain by inference from premises already known a priori. For example, the conclusion of an argument constitutes inferential a priori knowledge given that the premises in the inference are already known a priori.

[2] Kitcher, Philip, (1983), *The Nature of Mathematical Knowledge*. Oxford: Oxford University Press.

[3] For Bob Hale, for example, what can be said about a priori knowledge could as well constitute a response to Kitcher. (*Abstract Objects*, 1987, Oxford: Basil Blackwell Ltd, pp. 123-48.) See also Aaron Edidin's ""A Priori" Knowledge for Fallibilits", Philosophical Studies, vol. 46, pp. 189-98. Hale affirms:

fense of the notion of a priori knowledge to be a response to Kitcher's views. For instance, I agree with Bob Hale that "what can be said about a priori knowledge could as well constitute a response to Kitcher". Kitcher's position is very interesting because it is an example of a conception of an a priori infallible route while denying that mathematical knowledge is a priori.[4]

In this paper I shall evaluate what is considered Kitcher's classical conception of an a priori warrant in his book (1983). I will argue that his denial of the apriority of mathematical knowledge is unsound.[5]

Section 1: Kitcher's account of the notion of a priori knowledge

For Kitcher, as well as for many other epistemologists, an adequate general account of knowledge has to put some constraint upon how a state of true belief must be produced if it is to amount to knowledge. Kitcher's proposal is that a suitable constraint has to be concerned with the psychological processes which produce beliefs. Since his proposed constraint is on psychological processes, he calls his view "psychologistic". An example of a belief-producing psychological process is the process of "following a proof".[6]

Kitcher's general analysis of the concept of knowledge is:

> "(1) X knows that p if and only if X believes that p and X's belief that p was produced by a process which is a warrant for it." (p. 17)

Kitcher specifies that (1) is just the starting point for a general characterization of the concept of knowledge.[7] Kitcher introduces the term "warrant" to refer to

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> But it is clear that defense [of the notion of a priori knowledge] is much needed, not only because skepticism about the applicability of the notion of *a priori* knowledge has not always, or even typically, been motivated by adherence to a specifically *causal* view of knowledge, but also because at least one writer [Kitcher] has developed a notion of knowledge *a priori* within a framework of a broadly causal epistemology whilst arguing that our mathematical knowledge cannot be of that ilk. (Hale, p. 125)

[4] Kitcher's book constitutes an attack on the thesis that mathematical knowledge is a priori.

[5] I will not discuss the issue of the apriority of logic in this paper.

[6] Kitcher, pp. 42-3.

[7] But how can the right hand side of (1) be logically equivalent to its left hand side? In the left hand side of (1), truth is implied since knowledge entails truth. In the right hand side, there is no talk of truth. It appears then that in order for the right hand side to be logically equivalent to the left hand side, a warrant has to imply the truth of p.

However, if a warrant implies the truth of p then it has to be an infallible warrant. And if so, then the concept of an infallible warrant won't be distinctive of the concept of a priori knowledge,

those processes which produce belief "in the right way". The notion of what it is for a process to warrant a belief is deliberately vague to leave open which of the various psychologistic accounts is preferable. The notion of "warrant" or "justification" makes possible a distinction between different ways we acquire knowledge, i.e., a priori or a posteriori.

Kitcher's analysis of the notion of a priori knowledge is:

> "(2) X knows a priori that p if and only if X knows that p and X's belief that p was produced by a process which is an a priori warrant for it.
>
> (3) α is an a priori warrant for X's belief that p if and only if α is a process such that, given any life e, sufficient for X for p,
>
> (a) some process of the same type could produce in X a belief that p;
>
> (b) if a process of the same type were to produce in X a belief that p, then it would warrant X in believing that p;
>
> (c) if a process of the same type were to produce in X a belief that p, then p." (p. 24)

How do we get the a priori from this characterization? The a priori is attached to the warrant. Given that X has the necessary concepts to entertain p, an a priori warrant for X for p could produce belief that p, warrant p, and, given that the warrant produced belief that p, then p is true. 'If someone knows a priori that p then she could know that p whatever sufficiently rich experience she had had (sufficiently rich experience to entertain the proposition to be known).'[8]

Again, given that a subject is equipped with the necessary concepts to entertain a belief to be known, p, a process of the type in question would, were it to produce the belief that p, produce a warranted and true belief. Any belief produced by an a priori warrant is knowledge. It is assumed that the knowers in question are humans and that their intellectual capacities remain fixed across all their possible lives.[9] Kitcher's general problem becomes: how can there be warrants that always produce knowledge? Kitcher tries to show that there can be no a priori mathematical knowledge. For Kitcher, a priori grounds for belief must be such

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as Kitcher maintains. Kitcher is trying to provide here a general analysis of the concept of knowledge.

I think this problem may be explained by the fact that (1) is allegedly only providing a starting point in this analysis. Unfortunately, Kitcher does not elaborate on (1) in his book.

[8] P. 24.

[9] PP. 26-7

that it could not be rational to regard them as insufficient to justify belief under any experiential context. Mathematical grounds allegedly do not satisfy that condition on a priori grounds.

Section 2: On Kitcher's thesis that a priori knowledge is incompatible with revision

For Kitcher, a priori beliefs must be unrevisable because they are produced by a priori warrants. A priori warrants only produce and warrant true beliefs.[10] Mathematical beliefs cannot be warranted by a priori warrants because some of the mathematical beliefs we take ourselves to be justified in believing are false and their justifications faulty.

Kitcher claims that routes to mathematical knowledge can always be upset by unkind experience,[11] they can lead us astray, so they cannot be a priori for that reason. Kitcher's point is twofold: routes to mathematical knowledge are not infallible, so they are not a priori; and they can be upset by unkind experience making things even worse for the defender of mathematical routes as being a priori.

> Experiences which cast doubt on the accuracy of the book (by appearing to expose errors in many 'theorems', let us say), and in which eminent mathematicians denied the conclusion, *would interfere with the ability of the process to warrant the belief.* If I check through the proof in a book, thinking I see how the inferences go, and if the proof is very complex, then, under the circumstances in which there is weighty evidence against both book and theorem, it would be unreasonably arrogant and stubborn of me to form the belief.[12]

Kitcher understands the term "experience" as the knower's total sensory experience; and "independent of experience" is understood counterfactually: knowledge that could have been obtained by a process of the same type that actually produced it, no matter what the course of the subject's sensory experience is, provided it was sufficient to allow him to acquire the relevant concepts required to grasp the proposition in question.

Kitcher claims that the "processes which apriorists take to generate our mathematical beliefs would be unable to warrant those beliefs against the background of a suitably recalcitrant experience".[13] Kitcher explains:

10 P. 24.

11 Unkind evidence is evidence against a warrant or the belief it warrants.

12 P. 43; my emphasis.

13 PP. 88-9.

> If apriorists are to escape this criticism on the grounds that the analysis of a priori is too strong, then *they must allow that it is not necessary for an a priori warrant to belong to a type of process members of which could warrant the belief in question given any sufficient experience.* To make this concession is to abandon the fundamental idea that a priori knowledge is knowledge which is independent of experience. The apriorist will be saying that one can know a priori that p in a particular way, even though, given appropriate experiences, one would not be able to know that p in the same way. But if alternative experiences could undermine one's knowledge then there are features of one's current experience which are relevant to the knowledge, namely, those features whose absence would change the current experience into subversive experience ... To reject condition (3b) ... would be to strip apriorism of its distinctive claim.[14]

Kitcher argues that if apriorists try to respond to his criticism alleging that his analysis of a prioricity is too strong, then they have to accept that it is not necessary for an a priori warrant to belong to a type of process which could warrant belief provided there are sufficient experiences for the acquisition of the conceptual repertoire needed for a priori knowledge. For Kitcher, if the apriorist makes this concession, he will be abandoning the crucial claim that a priori knowledge is independent of experience. A priori knowledge would be dependent on experience. The apriorist will find himself in the following awkward position: that one can know a priori that p in a particular way, even though, given unkind experiences, one would not be able to know that p in the same way. Now if there were alternative experiences that could undermine one's a priori knowledge that p, then there are features of one's actual experience which are relevant to the a priori knowledge, that is, those features whose absence would change the current experience into subversive experience. If the apriorist rejects condition (b) - that is, if a process of the same type were to produce in X a belief that p, then it would warrant X in believing that p - of Kitcher's analysis, he would be stripping a priori knowledge of its distinctive character.

Section 3: The issue of long proofs or calculations

Kitcher discusses whether long proofs can be a priori in his sense. He argues that long proofs cannot generate a priori knowledge of their results.[15] Kitcher relies at this point on Descartes' view [16] that long proofs are problematic sources of

14 Ibid; my emphasis.

15 PP. 40-3.

16 Descartes, Rene, (1967), Philosophical Writings, Rule VII. Edited by E. S. Haldane and G. R. T. Ross. Cambridge University Press, vol. 1, p. 19. I won't discuss whether Kitcher's interpretation of Descartes's view is correct.

knowledge since we cannot apprehend them all at once. According to Kitcher, Descartes' proposal is to go over the proof several times until we are able to apprehend it as a whole, all at once, and do not have then to rely upon the memory of having justified its earlier steps. Obviously, there is going to be a limit, an upper bound, to what we can achieve by this method. Then, what happens with our knowledge of truths that exceed this upper bound? Kitcher's response is that there can't be any a priori knowledge obtained via long proofs and that there are no rules of inference that preserve a prioricity. That is, there are no rules of inference such that if we start with premises supposedly known a priori, we are assured of ending up with statements known a priori. For Kitcher, the situation is even worse: our a priori knowledge of the premises, our starting points in the proofs, would be lost. The reason is the following: since we cannot apprehend the proof all at once, there is a switch of grounds from an a priori warrant in the beginning for the knowledge of the premises to knowledge based upon the memory of having followed such a warrant. Knowledge based upon memory can provide a reliable warrant, but not an a priori warrant. Kitcher concludes that there could be no rules of inference that could preserve a prioricity since, if there were any, we could construct proofs of arbitrary length whose results, if their premises where known a priori, would be known a priori as well.

Section 4: Knowledge obtained by "non-empirical processes"

Kitcher clearly allows that mathematical knowledge can be obtained by non-empirical processes. But he thinks that it is important to realize that though these processes are considered non-empirical they are not to be considered a priori warrants. These processes fail to come up to his standards for being a priori warrants.

One of the non-empirical processes that Kitcher considers was proposed by Gödel.[17] Gödel's proposal was that mathematical knowledge can be generated by a sort of non-sensory apprehension - Gödel calls it "mathematical intuition" - directed on the platonistically conceived abstract subject matter of mathematics. Kitcher asks: Why might someone who believed that we had such a faculty be led to think that the knowledge which it generated was a priori? Because mathematical intuition is a non-empirical process. Anyone who confuses non-empirical processes which actually warrant belief with a priori warrants will read Gödel as upholding a priorism. Other examples of non-empirical processes which engender belief are following proofs or computations.

17 Gödel, Kurt, (1984), "What is Cantor's Continuum Problem". Reprinted in Benacerraf and Putnam (eds.), The Philosophy of Mathematics, Englewood Cliffs: Prentice Hall, pp. 470-85.

Kitcher does not explain what he means by a "nonempirical process". Nonetheless, he seems to be committed to the following theses:

> A. Certain belief-producing processes (for example, following proofs) are nonempirical processes. They may be significantly contrasted with empirical processes which engender belief mainly because they can be available independently of experience.
>
> B. Some of these nonempirical belief-producing processes can generate knowledge (produce true warranted belief).
>
> C. Some mathematical knowledge is generated by such nonempirical processes.

For Kitcher, while the absence of certainty is compatible with knowledge, it rules out a priori knowledge. Kitcher rejects Kripke's claim that a priori knowledge does not necessarily involve being certain.[18] The notion of "certainty" in question is left unexplained. Kitcher argues that the process of following a proof may give us knowledge - nonempirical - of its conclusion, but not a priori knowledge, because we can conceive empirical circumstances in which it would be irrational to cling to the proof. In such a situation, our having followed the proof would not be enough for knowledge. In order for the process of "following a proof" to be an a priori warrant, it must always produce knowledge.

Section 5: Some remarks on Kitcher's views

I will be identifying Kitcher's most vulnerable points in this section.

What does Kitcher mean by "belief" when he affirms that unkind experiences "would interfere with the ability of the process to warrant the belief"? Is it the psychological state of believing or the objective belief, the proposition believed? This is important when we try to figure out what unkind experiences are interfering with.

On the one hand, there is the psychological reading of the term "belief". According to this reading, Kitcher's point is a simple one: if there is strong empirical evidence against a warrant for p then one should not believe p (the proposition).

18 Kitcher, p. 43. The reference to Kripke is: Kripke, Saul, (1980)S, Naming and Necessity. Cambridge, Mass: Harvard University Press, p. 39.

Kripke affirms:

> Something can be known, or at least rationally believed, a priori, without being quite certain. You've read a proof in the math book; and, though you think it's correct, maybe you've made a mistake. You often do make mistakes of this kind. You've have made a computation, perhaps with an error. (ibid)

Suppose I think have followed a long and complicated proof that p. I find out later on that mathematicians think the purported proof is flawed. Given that I am not a mathematician, I should not believe p. Then since one should not believe the belief, one cannot know the belief (the proposition). It is a necessary condition for knowledge that we believe the belief. It is in this qualified sense, that a priori knowledge is not completely independent of experience, and in that Kitcher is right.

However this reading would be a weaker interpretation of Kitcher's position. Since to believe the proposition involved is a necessary condition for knowledge of any sort, it does not involve the distinctive character of a priori knowledge. And Kitcher proposes an account of the latter, so he ought to take into consideration the distinctiveness of a priori knowledge. What I mean is that since any knowledge is dependent on experience in this way, if there is a priori knowledge, as Kitcher thinks there is, its characteristic "independence of experience" has to be accounted for in a way which respects such experience dependence.

Moreover, if experience always has this disturbing role - I should specify that what experience can always primarily disturb is our psychological act of believing and indirectly the proposition believed - even when it is misleading experience,[19] that ought to show *to Kitcher* that there cannot be independence of experience and, therefore, there can be no a priori knowledge. But Kitcher insists that there is a priori knowledge. Actually I think that Kitcher goes back and forth between the conception of revision as always possible even when it is the wrong thing to do, as in the case of revision, for example, being called for by misleading experience, and revision as the right thing to do on the occasion. But actual knowledge in general is only compatible with revision for the wrong reason and incompatible with revision for the right reason (more below).

On the other hand, Kitcher seems to be talking about the proposition believed as well as the psychological act of believing. Thus, another explanation of "the belief" in the phrase "would interfere with the ability of the process to warrant the belief" is that it is the proposition believed, so that, according to Kitcher, a priori warrants must be "ultra reliable", that is, they must always, in all counterfactual situations in which they are invoked, produce true beliefs (not only beliefs but always beliefs that are true). According to this interpretation, for Kitcher, it is

Kitcher is reacting to this passage of Kripke. Neither Kripke nor Kitcher explain what they have in mind when they talk about the notion of certainty. Kitcher's assertion that the absence of certainty is compatible with knowledge, but not with a priori knowledge is problematic. Knowledge is only compatible with probability of 1. Feelings of sureness is another matter.

19 Kitcher particularly stresses the role of misleading experience on p. 84 of his book.

not possible to separate the issue whether a belief has been acquired by an a priori warrant, and that belief's being knowledge. A priori warrants are "ultra-reliable" because they always *infallibly* produce knowledge by their means. (Note that I use the terms "ultra-reliable" and "infallible" interchangeably for that reason.)

Someone could ask: how "infallible" is involved in (3c)? Remember, "(3c) if a process of the same type were to produce in X a belief that p, then p."[20] Of course, the term "infallible" does not appear in (3c). It is involved in the sense that, for Kitcher, it is a necessary condition for processes to be a priori that they will *always* end producing true justified beliefs. That is, an a priori process is such that it produces justified belief and it is always the case that that justified belief is true. Isn't such a process infallible given that a priori knowledge is true justified belief a priori? Note that condition (3c) is derivable from (3b). (3b) imposes a condition on warrants to be a priori, namely, that under no experiential background in which they are available, can we regard them as insufficient to justify belief that p. A priori warrants always warrant belief.

A corresponding distinction can be drawn between the ability of the process to warrant belief, the proposition to be known, and the ability of the process to sustain my act of believing. The first is an epistemological matter; the second a matter of psychology. Kitcher's thought is that not only experience but even misleading experience can affect my ability to believe the proposition (the belief), so a priori knowledge cannot be independent of experience. In this sense, even misleading experience is always capable of interfering with my a priori knowledge, given that *to believe* (the psychological state of believing) is a necessary condition for knowledge.

The underlying assumption in Kitcher's argument is that if a priori warrants were not ultra-reliable, then "experience" could undermine a priori knowledge, and that would show that the latter is not independent of experience and, therefore, by definition, not a priori at all. It is interesting to note that what Kitcher would have needed is that *some* a priori warrants must be ultra reliable in order to have a priori knowledge in the first place. It is also assumed that only experience could undermine alleged "a priori" warrants. Kitcher does not consider the epistemic possibility of a priori grounds being undermined by a priori reasons. He does not allow for that possibility because he thinks that a priori warrants cannot be undermined in any way. I think that since Kitcher constructs the term "experience" very broadly, even if he could accept that there could be a priori reasons that could undermine an a priori warrant or its result, he would still characterize

20 Kitcher, p. 24.

this case as the *experience* of having a priori reasons to reject an a priori warrant or its result.

Kitcher explains that the aforementioned experiences, like those which put into question the accuracy of proofs by appearing to show errors in many of the theorems, "would interfere with the ability of the process to warrant belief". I propose that a distinction should be drawn here. We want to distinguish between (a) a warrant that is good from (b) a warrant we believe to be good. A warrant, if it is a good one, has to warrant the belief. That is why it is a good warrant. (A warrant warrants belief that p if it gives us a pretty good reason to believe that p.) In the case when our warrant is the possession of a proof that p, that the proof that p warrants p is not a matter of degree; it is an absolute yes or no; it does or it does not warrant p, contrary to what the phrase "would interfere with the *ability* of the process to warrant belief" appears to convey (my emphasis). A separate matter is our believing that a warrant is a good one. *To think* that we have a proof, for example, is a matter which admits of degree. We can be more or less confident in believing that our warrant is sound. On the other hand, to think *rightly* that we have followed a proof that p is surely *not* a matter of degree. Kitcher is obviously right in the sense that we have to believe the proposition in order to be able to know it. Also, our belief that p would involve the accompanying belief that our warrant for it is a good one; otherwise we won't count ourselves as being justified in believing the proposition in question, let alone knowing it.

It is wrong to think that we have to be very confident about this second-order belief - i.e. the belief that the warrant is good - in order to be justified in believing that p, and in order to know a priori that p, if one knows that p. (Note that the belief that a warrant is good is also accompanied by other collateral beliefs like that I have understood the reasoning in its entirety.) The matter is one of degree here since our necessary belief that the warrant is good, our having confidence in the warrant, admits degrees. Although we ought to attach to the warrant some *appropriate* degree of confidence (sureness) in order to be able to take ourselves as in possession of pretty good reasons which justify our believing a proposition. But, again, this is independent from the fact that does not admit of degrees: namely, that the warrant is good or not as a matter of fact.

As already said, Kitcher is right in claiming that empirical considerations can undermine a priori knowledge in the sense that they can undermine our ability to believe that a warrant is sound and, consequently, our ability to believe the proposition which it (presumably) justifies. However, we still can believe, or even know, a proposition without being sure that our warrant for it is good.

Kitcher ignores the fact that a warrant could be good, regardless of our thinking that it is; a warrant could be objectively an a priori warrant in his (strong) sense even if we don't know it. I believe that the reason why Kitcher ignores this fact is because he simply cannot accept it. He understands the notion of a warrant - of any sort - "psychologistically". In the case of an a priori warrant, since the a priori warrant is a mental process, like following a proof, and not the proof itself, we have to believe that the warrant is a good one in order for the warrant *always* to produce belief, that is, to make us believe the belief that p. Now we have seen that we don't have to be *totally* confident about the second-order belief that our warrant is sound in order for the warrant *always* to make us believe the belief that p.

For many philosophers the grounds for p, in the case of inferential a priori knowledge that p, should be the proof itself. But Kitcher is right that our grounds for believing a proposition p, or knowing p, in the case of inferential a priori knowledge (in Kitcher's view: in the case of inferential "nonempirical" (≠ "a priori") knowledge) that p should be the process of following a proof. A proof becomes a warrant by its being appreciated. And we can only appreciate a proof by following it. What is wrong is to think like Kitcher that the experience independence of a priori knowledge cannot leave room for the experience dependence necessary for the truth that we have followed a proof that p. For Kitcher, since we can make mistakes in following proofs (or in constructing them), then "following a proof" cannot be an a priori ground for believing that p. However, if the process of "following a "proof", I should say, a sequence of formulas, is sufficient for knowledge, (remember that Kitcher considers the process of "following a proof" a non-empirical process which sometimes can engender knowledge) the sequence has to be a proof-token. Then, why "following a proof" cannot be an a priori ground for believing that p if when we know by their means it is implied that we did not make a mistake in following the proof?

Is the justification for p the same as the justification for our belief that p? I do not think so. I want to distinguish between the conditions for the existence of a justification - for the existence of a routine, on paper or in thought, which someone can rightly take to justify him in believing that p- and the conditions for the existence - possibility of - a calculation, for example, which justified some arithmetical proposition entirely concern -one would suppose - what is permitted by the rules of arithmetic, and thus involve no element of contingency; but rightly thinking oneself justified by what is in fact a correct calculation may well involve empirical presuppositions about oneself and the prevailing circumstances (that one is not confused, or too drunk, and so on.)

This distinction is valid and important. Missing it may be one thing that has motivated philosophers to think that a priori knowledge distinctively entails infallibility, for instance.

Accordingly, I distinguish between the role some empirical assumptions can have in our a priori knowledge:

(1) In justifying the conclusion of a proof in the reasoning itself

(2) The role they have in the reasoning for the conclusion that "I have a proof".

The statement that "I have a proof" is an empirical statement and depends for its truth on empirical assumptions like that I did not make any mistake in carrying out the proof, [there is no mistake in the proof,] for example. My knowledge is probabilistic in that sense and it is empirically defeasible.

On the issue about the incompatibility between a priori knowledge and revision, let me observe briefly that it is incoherent as it stands. Of course, Kitcher is right about the incompatibility of a priori knowledge with revision. However Kitcher arrives at this true conclusion by the wrong reasoning. According to Kitcher, a priori knowledge cannot be compatible with revision. By contrast, Kitcher thinks that empirical knowledge is so compatible. But this is a serious confusion, again, because knowledge of *any* sort is incompatible with revision. Revision is compatible with justification and belief (not with knowledge and truth). Only warrants and beliefs are revisable.

This confusion is a pervasive problem which creates all sorts of misunderstandings. For now I will just say that the candidates for revision are beliefs, in particular, claims to knowledge (i.e. they are of the form: "I know that p"), and justifications (warrants). We change our minds about beliefs and the warrants which justify them. We may erroneously revise our beliefs and its warrants. In a *weak* sense, knowledge is compatible with revision since we can change our minds about what is in fact true and we are fully justified in believing. In another *strong* (normative) sense, knowledge is incompatible with revision because if we mistakenly revise, we cease to know since we cease to believe to be justified.

Two cases are to be distinguished: (1) when revision is the right thing to do; and (2) when we mistakenly think that revision is the right thing to do. When we have knowledge then revision in the second sense is the only thing possible and then we end up not knowing or ceasing to know. But then Kitcher is wrong to

think that the negation of (1) is only true in the case of a priori knowledge. In other words, when we have actual knowledge, then (rightly) revising is not possible in any case.

To conclude: The mistake Kitcher made is to think that an a priori warrant has to be ultra-reliable (infallible) to be a priori. However, Kitcher does not consider the idea that *some* a priori warrants may prove to be infallible, what Kitcher called "ultra-reliable", even if indirectly as results of the correct implementation of infallible methods.[21] Kitcher's mistake is that he does not distinguish between methods and warrants and, therefore, demands of warrants infallibility in order to be a priori when this property is (*primarily*) of *methods* and *not* of warrants.

The distinction between "methods" and "warrants" is very important. Let me briefly say that a method is a cognitive routine which can be performed correctly or incorrectly on the occasion, and we get warranted belief (or not) as a result. A warrant is a particular implementation of a method. For example, "constructing proofs" is a method; the particular proof I construct is my warrant for its conclusion. The notion of infallibility applies primarily to *methods*, and only *derivately* to warrants. This is a very complex issue that I tackle elsewhere.

It is important to appreciate that Kitcher's view on apriority is sensitive to the difficult issue of the unrevisability/infallibility in connection with a priori knowledge. Nonetheless, Kitcher's rejection of mathematical knowledge as a priori is an undesirable and mistaken consequence partly due to his oversight of the distinction between methods and warrants. The foregoing discussion points to the unsoundness of his argument for the denial of the apriority of mathematical knowledge.

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21 See my work "Some consequences of a notion of infallibility", forthcoming.

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