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KEPLER'S *HARMONICE MUNDI*: DEAD END ROAD OR FINAL THEORY?*

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I. Introductory remarks

Kepler opens the final book V of his work *Harmonice Mundi* (1619, hereafter '*HM*') with the passage - I omit parts of Kepler's very long, Latin style sentences - :

> The discovery which I foretold twenty two years ago, when I first discovered the five solid figures between the celestial spheres [Kepler here refers to his early work *Mysterium Cosmographicum*(1596)][1] ... the discovery for the sake of which I applied the best part of my life to astronomical studies, I visited Tycho Brahe, and I chose Prague as my seat; that discovery at last, with God the Best and Greatest, who had inspired my thought and had aroused this mighty ambition, also prolonging my life and the vigor of my talents ... on the completion of my previous work in the province of astronomy to a sufficient extent, at last, I say, I have brought that discovery into light, and have most truly grasped beyond what I could ever have hoped: That the whole nature of harmony, to its full extent, with all its parts, ... is to be discovered among the celestial motions. ... during this intermediate period, in which the extremely laborious restoration of the motions held me in suspense, my appetite was particularly intensified and my purpose stimulated by the reading of the *Harmony* of Ptolemy ... There I found unexpectedly, and to my great wonder, that almost the whole of his third book was given up to the same study of the celestial harmony, one thousand five hundred years before. Yet at that period much was still lacking in astronomy ... Now, eighteen months after the

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* A previous version of this paper was given at UPR in March 2000.

[1] hereafter '*MC*'; cf. n. 8. There was a second edition in 1621, with interesting comments by the author.

> first light, three months after the true day, but a very few days after the pure Sun of that most wonderful study began to shine, nothing restrains me; it is my pleasure to yield to the *inspired frenzy* ...If you forgive me, I shall rejoice; if you are enraged with me, I shall bear it. See, I cast the die, and I write the book. Whether it is to be read by the people of the present or of the future makes no difference: let it await its reader for a hundred years, if God Himself has stood ready for six thousand years for one to study him.[2]

The "inspired frenzy" refers to the detection of the "harmonic law", later known as Kepler's *third law* of planetary motion: whereby the planets' periods, T, are linked to their mean distances from the sun, r, by the proportionality

$$T^2 \sim r^3.$$

The first law had stated that each planet moves around in an ellipse with the sun in one of its foci, while the second, the so-called area law, links the varying speeds of a planet to its varying distances from the sun. These two laws were already stated ten years earlier in the *Astronomia Nova* (1609).[3] They differ considerably in character from the third law: they are applicable to the planets *individually*, while the third law relates the planets to each other: they all have the same ratio T^2/r^3.

The three laws named after Kepler were later considered as his lasting contribution to astronomy. What goes with them in Kepler's own perspective, however, was pretty much forgotten after Kepler's death (1630).[4] This is esp. true for what the third law meant to Kepler himself: he regarded it as the finally found cornerstone of the harmonic system of the world, while in historical "reality" it became the decisive starting point for the first modern universal theory, Newtonian gravitation. Thereby Kepler's constant T^2/r^3 turned out to be a *universal constant*, which can essentially be identified as the constant of *gravitation*.[5]

What Kepler had in mind, however, was not general laws, but the *design of a concrete system*: the finite cosmos of the Sun and its

[2] AITON et al. (transl.), 389-391; my emphasis.

[3] Cf. the somewhat fuller statement of Kepler's laws in my 'The Structure of the Copernican Revolution', *this journal* **77** (Jan. 2001), pp. 7-24 (hereafter referred to as '*CR*), sec. I..

[4] For the (poor) reception of *HM* cf. STEPHENSON [94], ch. X, 'Conclusions'.

[5] cf. *CR*, sec. II.

planets. He looked for this design in terms of symmetry principles, harmonic relationships etc., not general laws in the modern sense. But, being a well-trained mathematician and scrupulous empiricist, he hit upon most remarkable results in the modern sense of physical science. In his search for divine design, his lasting results were more or less only by-products. This clash between a historiography of scientific progress and Kepler's own perspective of a final "theory", or rather system, is what I am going to investigate somewhat further - thereby hopefully illuminating also more generally the conditions of his *creativity*.

II. Kepler at work

Most people know Kepler only as an astronomer. But he worked in many fields. He had entered the University of Tübingen and chosen as a subject theology in order to become a Lutheran minister. And in a way he remained a theologian all of his life. During the general studies - that he, like all students, had to go through - he got acquainted with Copernican astronomy, taught to him, in private lessons, by his teacher MAESTLIN, one of the few Copernicans of late 16th century. Young Kepler was very attracted to Copernicus' heliocentrism for theological and philosophical reasons; evidently he liked the *Platonistic* flavor of heliocentricity.[6]

But Kepler went much further than Copernicus with his Platonism. Shortly after he had accepted a job as "mathematician" in Austria - actually his main task was teaching astrology, casting horoscopes, and writing astrological calendars - he wrote his first book, the already mentioned *Mysterium Cosmographicum* (1596),[7] in which he not only proposed Copernicus' heliocentric system, but reasoned for it in a very special way: like Plato in *Timaeus* Kepler offered an explanation of the cosmos by attributing certain geometrical ideas to the Creator. By this he answered to three questions, which he explicitly stated:

1. Why are there just 6 planets?

2. Why do the "spheres" of the planets have just

[6] Kepler, like Copernicus before him, almost worshipped the Sun.

[7] '*Mysterium Cosmographicum*' is only part of the title, which reads, in DUNCAN's translation: *Forerunner of the Cosmological Essays, which contains THE SECRET OF THE UNIVERSE*. (Cf. also the next note.) Kepler thus indicated that he planned to continue this kind of work.

the proportions, which they actually have (according to Copernicus)?

3. Why do the planets have just the periods they actually have?[8]

What is especially remarkable of this set of questions, is that they ask for (what we would call) *contingent* explanations of *contingent* facts, i. e. for something that is not likely to refer to natural laws. The first two questions - or the respective questions for all planets, including the ones still to be detected - were later answered "historically", namely by "telling a story" how the planetary system might have come into being in accordance with Newton's theory of gravitation (Laplace & Kant in the 18th century). - The third question is somewhat different in character since there was already a certain rule to be explained: that the planets are the slower the more distant from the center they are. Kepler tried hard - and, in 1596, in vain - to find the exact relationship between distances r from the Sun and periods T. Evidently:

(1) $T \sim r$

would make the planets' velocities depend *too little* on their distances from the center; in fact (1) would mean that they all have the *same* velocity, contrary to Kepler's intuition that the Sun propels the planets around and consequently would be less efficient for the planets further away. Also (1) was grossly mistaken empirically. - The next try would naturally be

(2) $T \sim r^2$

But this would make T depend *too much* on r, given the Copernican data. It was only much later that Kepler hit upon what might be regarded as the next step to be taken:

(3) $T \sim r^{3/2}$

or $T \sim r^3$,

[8] In a less prosaic form Kepler mentions these issues already in the rest of the lengthy title of his work (cf. the previous note), which runs, again in Duncan's translation: *On the Marvelous Proportion of the Celestial Spheres, and on the true and particular causes of the number, size, and periodic motions of the heavens, Established by means of the five regular Geometric solids.*

i. e. the famous *harmonic law* of *Harmonice Mundi*, which Kepler was so enthusiastic about, as quoted at the beginning.[9]

That Kepler did not find the third law earlier is nothing one should blame him for. He did not - and, historically could not - ask his third question in the manner I just did: namely the question what the specific mathematical form of

$$T = f(r)$$

is. He rather worked through the list of planets one by one and correlated, for pairs of neighboring planets, the *increments* or *decrements* of T with the *decrements* or *increments* of r.[10] These *ad hoc* calculations, however, did not lead to any satisfactory result. On the other hand Kepler managed to find a definite solution to his first two questions (within the boundaries set by the astronomical data he had). The solution he found is the famous model (as we would say) of nested *Platonic solids*[11] which answered both questions at the same time: Since the polyhedra span the spaces between the planetary spheres, there have to be just six planets as postulated by Copernicus (as opposed to Ptolemy according to whom there are seven planets, including Sun and Moon, excluding Earth). And the distances of the planets from the center are approximately those calculated by Copernicus. Kepler never gave up this early idea of his, only modified it, i.e. laid out more sophisticated models built on the basis of his first one. He repeated this Platonic doctrine not only in his cosmological works: *Harmonice Mundi* (1619) and the second edition of *Mysterium Cosmographicum* (1621), but also in the *Epitome astronomiae Copernicanae* (1618-21), a Copernican astronomical text-book, so to speak.

To summarize Kepler's main efforts of the *MC*: the number and the distances of the planets - *static* features of the cosmos, that is - were explained by (divine) *design*, while with respect to the third, *dynamical*, question young Kepler essentially had to quit. - But he never gave up.

[9] By the way, when Kepler eventually found (3) he was by no means sure that (3) would be *exactly* true; it seems he had become sceptical whether there really *is* an *exact* dependency. Cf. STEPHENSON [94], ch. I, p. 7.

[10] Cf. Stephenson [94], ch. V, for a reconstruction of Kepler's rather tricky calculations.

[11] Cf. CROMWELL [97] for geometrical and historical details, for Kepler esp. ch. 4. - In *MC* Kepler reports how astrological speculations lead him to the polyhedral hypothesis (Preface, transl. Duncan 62 ff); cf. Stephenson [94], ch. V.

By the end of the 16th century he tried to improve his system, both theoretically and empirically. Theoretically he developed, but did not publish, his first system of celestial harmonies.[12] And empirically he did all he could to gain access to the far more exact data, which Tycho BRAHE had accumulated in Denmark with the best available pre-telescopic instruments. When Brahe moved to Prague and became "imperial mathematician" to the Emperor Rudolph II, Kepler successfully applied for the position of Brahe's assistant, and when Brahe shortly afterwards suddenly died - fortunately, if I may say so - Kepler took the position of the imperial mathematician himself. Set on the task, by the late Brahe, to "conquer" the strangely behaving *Mars*, Kepler began what he seems to have regarded as an immense "detour" (cf. the citations from the beginning of book V of *Harmonice Mundi* above): a detour, i. e. with respect to what one would like to call Kepler's *harmonic research program*. In later astronomers' eyes, however, this "detour" was the real thing: Kepler's lasting contribution to astronomy, culminating in his *Astronomia Nova* of 1609.

While Kepler was working on the *Astronomia Nova* he got a copy of Ptolemy's *Harmonics* and was surprised how similar was his striving for a harmonic system of the world to that of Ptolemy 1500 years earlier - cf. the quotation from the beginning of book V of *Harmonice Mundi*.

But Kepler's system of harmonies was bound to be more complex than Ptolemy's, simply for the reason that, given the eccentricities of the ellipses, there were more data to fit into the harmonic schema, and, of course, also for to reason that Kepler's data - based on Tycho's observations - were far more exact than those of the Ptolemaic tradition and those of Copernicus as well. (Also there was a complication with respect to astrological aspects, which Kepler wanted to include in his system.)[13]

Kepler's way out was supported by music itself, so to speak: Just shortly before and in Kepler's time musical harmonic theory had been considerably expanded by incorporating *polyphony*.[14] For Kepler the historical coincidence of improved astronomical and musical findings was not by chance but by divine providence. God's ideas underlying

[12] Stephenson [94], ch. VI, has reconstructed this from Kepler's correspondence.

[13] cf. *CR* III, p. 17.

[14] One of the theoreticians of music whom Kepler studied was Vincenzo Galilei, father of the physicist Galileo; cf. FIELD [88], 117.

Creation had to wait to be detected by humankind until both developments had matured to a decisive point - and Kepler thanked God for being granted the privilege to find out the harmonical mysteries of Creation. That he, Kepler, finally hit upon the "*harmonic law*", could be explained, in Kepler's eyes, only by the highest source, i. e. by divine *revelation*.

Personally, the ten years between *Astronomia Nova* and *Harmonice Mundi* were extremely difficult for Kepler. In the eyes of his employer, the Emperor, the *Astronomia Nova* was only a preliminary step towards more practical goals: to establish exact astronomical tables for calendars, navigation, etc. After very laborious years Kepler finally came up to these expectations by the publication of the *Rudolphine Tables*. But not only this project kept him from his harmonical studies. Galileo's 'new stars', discovered with the just (re) invented telescope, frightened Kepler as possibly destroying his basic Platonic model of the cosmos. It came as a relief that the 'new stars' turned out to be merely satellites to Jupiter and thus did not severely disturb his system.[15]

Worse for Kepler than these and other scientific distractions from his harmonical projects was his economical and family situation. Difficulties accumulated when in 1618 not only the 30-Years-War broke out but Kepler's mother was definitely threatened to be convicted for witchcraft. Only repeated personal and legal interventions by Kepler prevented his mother from being severely punished.

III. Kepler's creative '*dream of a final theory*'

Kepler's work is a decisive part of the so-called *Copernican Revolution*; in fact, it is, in my eyes, the turning point of the "Copernican Revolution". (In this paper I am not questioning whether the development which we are used to call this way really is a revolution.[16]) This crucial role of Kepler's work depends on his peculiar *perspective* on astronomical and cosmological matters, which is neither that of Ptolemy or Copernicus, nor that of Newton, although it comprises

[15] Kepler's response to Galileo's *Sidereus Nunceus* (1610) was published already in the same year in his *Dissertatio cum Nuncio Sidereo*, Prague; cf. Field [88], 77-80. - In *HM* Kepler uses so-called rhombic polyhedra to account for Jupiter's satellites; cf. *loc. cit.* 110.

[16] For this issue, as well as for some of the following remarks, cf. *CR*.

elements of all three of them: As we have seen, Kepler aims at roughly the same as *Ptolemy*, namely at a system of the world which is *mathematically* correct, *physically* sound, *astrologically* illuminating, and *harmonically* satisfying. Of course, Kepler superseded Ptolemy in various respects, notably with regard to heliocentricity and polyphony, as well as in integrating the four fields just mentioned into one coherent system. In this way Kepler may be regarded as the one who finally fulfilled Ptolemy's "research programme".

Is there a methodological moral to be drawn for conditions for scientific creativity? Should researchers be allowed their own personal, even idiosyncratic perspective? What is it that turns scientists on, makes them creative? There is no general answer to these questions, I suppose. In Kepler's case I am quite sure that the strange co-ordinates of his world view, including his theology and his astrological inclinations, were a *conditio sine qua non* for his scientific creativity. But for sure, having extra-scientific ideas like Kepler did, is by no means sufficient for creative scientific work. A case in question is Kepler's contemporary Robert FLUDD. This English physician had developed a fantastic harmonic system of his own - pure superstition, I would say. Fludd was pretty influential at his time, and Kepler made considerable efforts to criticize Fludd, esp. on *methodological* grounds.[17] In more modern terms, I would say, the main methodological difference between the two is that Kepler, in spite of his Platonism, was a good *empiricist*, actively putting to test his conjectures where possible. (This could be exemplified especially well in his *astrological* work; but that is a complicated story in itself.)

In the preface to the English translation of *Harmonice Mundi* A. M. Duncan and J. V. Field write: "The *Harmonice Mundi* may fall short of its author's pretensions. Yet it presents a remarkable picture of the universe, composed according to the same methodology that produced the three laws of planetary motion; namely, a methodology in which hypotheses are built upon and confirmed by observations."[18] While this statement is not plainly false, it presents, in my eyes, a far too meager characterization of Kepler's methodology in *Harmonice Mundi*, exactly because its "hypotheses" do not merely relate to observations, but also, like a Janus head, to principles of mathematical beauty and divine

[17] Cf. Kepler's appendix to *HM*, book V.

[18] AITON et al., p. ix.

benevolence. The two authors proposed a somewhat fuller account just one page earlier: "Yet today his [Kepler's] world harmony is seen to possess an essential element of truth. ... Kepler's use of formal causes is in line with the modern physicists' use of symmetry principles ..." This is more to the point and worthy of further investigation. Is there, indeed, a methodological *rationale* in Kepler's work, which has some bearing on today's philosophy of science?

More specifically, I would like to ask: is the peculiar form of theory chosen by Kepler in his *Harmonice Mundi* still interesting for today's scientists? Given the fact that through the centuries since the publication of *Harmonice Mundi* its readers probably felt more and more alienated, this might seem a hopelessly misguided question. Yet I think there are certain parallels to 20th century physics - namely with respect to the (alleged) *finality* of Kepler's theory.

This term has recently received special prominence by Steven WEINBERG'S *Dreams of a Final Theory* (1993). As Weinberg points out, the "final theory" that he, like other physicists, is after, will *leave nothing to explain*. The possibility of such a theory may seem a miracle to the adherents of the wide-spread philosophical image of explanation according to which facts may be explained by laws, these in turn by laws of a higher level, and so on, i.e. the open-ended chain of explanations will leave us with never-ending why-questions. Why is Weinberg so confident that physics will come to an end? His hope lies in certain traits of advanced quantum mechanics. Some physicists/philosophers like C.F. VON WEIZSÄCKER connect the Copenhagen interpretation of quantum mechanics with a transcendental program in the sense of I. KANT: the basic principles of science are of an *epistemological* nature.[19] Finally (going backwards in time) Kepler claims to have traced cosmology back to the very principles of divine creation, comparable in character to PLATO's *Timaeus* (which Kepler, like many before him, regarded as an inspired commentary on *Genesis*).[20] Of course, these are only parallel *grosso modo*. I want to stress, however, that Kepler - as Plato before him and Kant and some modern philosophers of physics after him - doesn't just put forward *a priori* speculations but insists on the *empirical soundness* of his constructions.

[19] cf. e.g. von Weizsäcker [66].

[20] cf. Field [88], ch. I, esp. p. 1, n. 1.

Many researchers try to create the ultimate work in their respective field. Kepler is no exception in this respect. What may be exceptionable in his case is the fact that he seems to have been not so much interested in personal fame as in the fulfillment of a divine plan: God's revelation in Nature as understandable by sensible human beings. Several disciplines had to join in order to reach this goal: musical harmony had to be perfected, and the art of astronomy as well. Both happened, almost incidentally, around 1600. Kepler was chosen by divine providence to bring about the one perfection, that of astronomy, and combine it with the other one to form a coherent whole: the system of *HM*.

This seems to be the way, roughly, how Kepler felt about his role in the history of human knowledge. If Kepler had not been deeply convinced that there actually *exists* a harmonic structure of the cosmos, he hardly could have fought so intensely, and for no less than twenty years (1599-1619), to fulfill what he considered to be his calling. The obstacles, empirically and mathematically, were immense. If he had not had a *vision*, if he had merely followed a modern methodology of hypothetical deductivism, he could not have created the system he "found".

From a historical perspective, however, things look quite different. By coincidence, one would say, astronomical sophistication had just reached the appropriate level in Kepler's time: the data were *accurate* enough to conform to the complex structure of musical harmonies in the age of polyphony, and they were *crude* enough not to exhibit serious discrepancies.[21] Good luck for Kepler that telescopic astronomy was only just beginning, and probably a *conditio sine qua non* that further planets had not yet been detected.[22]

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[21] cf. Stephenson [94], ch. I, p. 8.

[22] *Uranus* was detected only in 1781. But, curiously enough, there are some hints that Galileo, without knowing, had in fact detected *Neptune*; cf. SWERDLOW [98], p. 258.

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